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ON (r, p)-CAPACITIES FOR MARKOV PROCESSES

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1. Introduction

For a general Markovian semi-group $\{P_t;\ t\geqq 0\}$ on a measure space, we consider the image $F_{r,p}$ of L_p-space of the r-th order Γ-transformation of P_t. Then $F_{r,p}$ gives rise to a set function $C_{r,p}$ satisfying certain properties of capacity (M. Fukushima and H. Kaneko [6]). When P_t is a symmetric operator on L_2-space, the capacity $C_{1,2}$ coincides with the capacity related to the Dirichlet space associated with P_t, and consequently, the set of zero $C_{1,2}$-capacity can be identified with the polar set of the Hunt process corresponding to P_t, if the latter ever exists ([5]). But as r or p becomes greater, the set of (r, p)-capacities zero become finer. For instance, when P_t is the heat kernels on R^n, the Γ-transformations of P_t are equal to the so-called Bessel kernels. Therefore, in that case, $C_{r,p}$ coincides with the Bessel capacity $B_{r;p}$ presented in [11], for which there exists no non-empty sets of zero capacity whenever $rp>n$ ([11]).

The purpose of this paper is to examine whether some basic theorems related to the Markovian semi-group $\{P_t;\ t\geqq 0\}$ can be refined, so that one may take the sets of $C_{r,p}$-capacity zero for various r and p as exceptional sets in the statement of the theorems. Assuming the analyticity of P_t, we shall show that two refinements (Theorem 1 in §2 and Theorem 3 in §4) of this kind are indeed possible. The first one is for ergodic theorem due to G.C. Rota [13], E.M. Stein [16] (which concerned m-a.e. statements) and due to M. Fukushima [4] (which concerned $C_{1,2}$-q.e. statement). The second is for the construction of a Hunt process which has been established by M. Fukushima [5] and M. Silverstein [14] in the case that $(r, p)=(1, 2)$ and P_t is symmetric and by S.C. Menendez [10] in a non-symmetric case. In §3, a refinement in the construction of a transition function will be presented.

In this connection, we mention the work of Y. Le Jan [8] who started with a general Markovian semi-group on an L_∞-space and constructed a Hunt process with exceptional set being related to a certain family of supermedian functions. While the above mentioned papers and ours start with a Markovian semi-group acting on an L_2-space or L_p-space, D. Feyel and A. de La Pradelle [3] started with the one acting on a Banach space of functions which are already refined in relation to a capacity. Further we mention a related work of N.G.