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A NOTE ON SOME PERIODICITY OF Ad-COHOMOLOGY GROUPS OF LENS SPACES

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1. Introduction

Let p be an odd prime and $q=p^r$. We choose a positive integer k such that the class of k in $\boldsymbol{Z}/p^2$ generates the group of units $(\boldsymbol{Z}/p^2)^\times$.

Let K^* be the K-cohomology theory and $K^*_{(p)}$ its p-localized theory. The Adams operation ψ^k on K induces a stable operation ψ^k on $K^*_{(p)}$. We denote by $K_{(p)}$ the spectrum which represents the $K^*_{(p)}$-cohomology theory. Since stable operations induce maps of spectra, we have the following cofibration of spectra

$$K_{(p)} \xrightarrow{1-\psi^k} K_{(p)} \longrightarrow C_{1-\psi^k}\,.$$

We define a spectrum Ad as $\Sigma^{-1}C_{1-\psi^k}$ and its associated cohomology theory Ad^*. When k is a prime power, the associated connective theory of Ad^* coincides with the cohomology theory defined by Seymour [9] and Quillen [8].

Let m and n be positive integers. We identify $\boldsymbol{Z}/m$ with the set of m-th root of 1 in C, and S^{2n+1} with the unit sphere in C^{n+1}. The complex vector space structure on C^{n+1} induces a $\boldsymbol{Z}/m$-action on S^{2n+1} and we define the standard Lens space mod m as $S^{2n+1}(\boldsymbol{Z}/m)$. As is well known, the standard Lens space $L^n(m)$ has a CW-complex structure

$$L^n(m) = \bigcup_{i=1}^{2n+1} e^i$$

and we denote its $2n$-skeleton by $L^n_0(m)$. Since the canonical inclusion $C^{n+1}\subset C^{n+2}$ induces a cellular inclusion $L^n(m)\subset L^{n+1}(m)$, we have a CW-complex $L^\infty(m)=\operatorname{colim} L^n(m)$. This space $L^\infty(m)$ is a classifying space $B\boldsymbol{Z}/m$ of $\boldsymbol{Z}/m$. We consider the case $m=p^r$. Main results are the following.

Theorem 1.1. *Let $M(n)=r+[(n-1)/(p-1)]$. For any integers i, j satisfying $i-j\equiv 0$ (mod $(p-1)p^{M(n)-1}$), there holds the following isomorphisms.*

$$\widetilde{Ad}^{2i}(L^n_0(p^r)) \cong \widetilde{Ad}^{2j}(L^n_0(p^r))$$
$$\widetilde{Ad}^{2i+1}(L^n_0(p^r)) \cong \widetilde{Ad}^{2j+1}(L^n_0(p^r))\,.$$