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ON Ad*-COHOMOLOGY GROUPS OF THE CLASSIFYING SPACES OF COMPACT LIE GROUPS

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Dedicated to Professor Nobuo Shimada on his sixtieth birthday

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1. Introduction

Let k be a positive integer and R a proper subring of $\underset{\sim}{Q}$ which contains k^{-1}. We consider the complex K^*-cohomology theory, the Bott element β and the Adams operation ψ^k. Since $\psi^k(\beta)=k\cdot\beta$ it induces the stable Adams operation ψ^k_{2m} in $K^{2m}(X; R)$, the R-coefficient K^*-cohomology theory. We consider the secondary cohomology theory [9] associated to the stable operation $1-\psi^k_*$ and denote the theory by $Ad^*(X; R)$. That is, Ad^* is a cohomology theory satisfying the following exact sequence

$$\cdots \to Ad^*(X; R) \to K^*(X; R) \xrightarrow{1-\psi^k} K^*(X; R) \to Ad^{*+1}(X; R) \to \cdots .$$

When k is a prime power, the associated connective theory of $Ad^*(X; R)$ coincides with the algebraic K-cohomology theory defined by Quillen [6] and Seymour [8]. We consider a compact lie group G and its classifying space BG. As is well known, $K^{2m}(BG; R)$ is a torsion free group and $K^{2m+1}(BG; R)=0$. So it is easy to see that $Ad^{2m}(BG; R)$ is a torsion free group. We denote the connected component of the unity by G_0, which is a closed normal subgroup of G and we consider the group of connected components G/G_0. Main result is the following.

Theorem. (i) *For any integer* $m\neq 0$, $\widetilde{Ad}^{2m}(BG; R)=0$.
(ii) *When* $|G/G_0|$ *is invertible in* R, *then* $\widetilde{Ad}^0(BG; R)=0$.

In section 2, we prove the Theorem for finite groups by considering their cyclic subgroups. In section 3, we prove the Theorem for compact connected Lie groups by considering their maximal tori. In section 4, we prove the Theorem for general compact Lie groups by considering their Cartan subgroups.

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