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ON 3-FOLD IRREGULAR BRANCHED COVERING SPACES OF PRETZEL KNOTS

Dedicated to Professor Minoru Nakaoka on his 60th birthday

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It is well-known that any orientable closed 3-manifold is a 3-fold irregular branched covering space of a 3-sphere branched along a knot. It is an interesting problem to know which 3-manifold can be a 3-fold irregular branched covering space of a given knot. In this paper we consider those of pretzel knots.

For the permutation group S_3 on $\{0, 1, 2\}$, let $a=(01)$, $b=(02)$, $c=(12)$, $x=(012)$, $y=(021)$. Then there are relations $a^2=b^2=c^2=1$, $ab=bc=ca=x$, $ba=ac=cb=y$. Especially, we remark the following relations:

$$\begin{array}{llll} aba^{-1}=c\,, & aca^{-1}=b\,, & axa^{-1}=y\,, & aya^{-1}=x\,, \\ bab^{-1}=c\,, & bcb^{-1}=a\,, & bxb^{-1}=y\,, & byb^{-1}=x\,, \\ cac^{-1}=b\,, & cbc^{-1}=a\,, & cxc^{-1}=y\,, & cyc^{-1}=x\,, \\ xax^{-1}=b\,, & xbx^{-1}=c\,, & xcx^{-1}=a\,, & xyx^{-1}=y\,, \\ yay^{-1}=c\,, & yby^{-1}=a\,, & ycy^{-1}=b\,, & yxy^{-1}=x\,. \end{array}$$

A knot group G has a Wirtinger presentation:

$$(x_1, x_2, \cdots, x_n; r_1, r_2, \cdots, r_{n-1}) \tag{1}$$

where each relator r_i indicates the relation form $r_i=x_{j(i)}^{\varepsilon}x_ix_{j(i)}^{-\varepsilon}x_{i+1}^{-1}$ $(\varepsilon=\pm 1)$ at a crossing as in Fig. 1.

Then a homomorphism from a knot group G to S_3 satisfies a condition as follows.

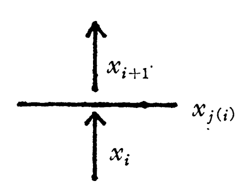

Fig. 1

Proposition 1. *Let the above* (1) *be a Wirtinger presentation of a knot group G. Then a homomorphism h from G to S_3 satisfies one of the followings.*

(i) $h(x_i)=a$ *(or b, c)* $\quad (i=1, 2, \cdots, n)$,

(ii) $h(x_i)=x$ *(or y)* $\quad (i=1, 2, \cdots, n)$.

Proposition 2. *Let* $(x_{11}, \cdots, x_{1n_1}, \cdots, x_{m1}, \cdots, x_{mn_m}; r_1, \cdots, r_k)$ *be a Wirtinger*