Kawai, O. and Miyanishi, M.
Osaka J. Math.
23 (1986), 207–215

SPECIALIZATIONS OF COFINITE SUBALGEBRAS OF A POLYNOMIAL RING

Dedicated to Professor Hirosi Nagao on his sixtieth birthday

OSAMU KAWAI AND MASAYOSHI MIYANISHI

(Received December 14, 1984)

1. Introduction. Let K be a field of characteristic zero and let $R_K := K[x, y]$ be a polynomial ring in two variables over K. A normal K-subalgebra A of R_K is said to be *cofinite* if R_K is a finite A-module with the canonical A-module structure. In the case where K is an algebraically closed field, we know the following results:

(1) If A is regular, A is then a polynomial ring in two variables over K; see [3] and [8].

(2) If A is singular, then there exist a polynomial subalgebra R'_K and a finite group G of linear K-automorphisms of R'_K such that $A=(R'_K)^G$ and G is a small subgroup of $GL(2, K)$; see [4] and [10].

In the present article, we shall show that the structures of normal cofinite subalgebras A of R_K are invariant under specializations, provided the quotient field extension $Q(R_K)/Q(A)$ is a quasi-Galois extension; see Definition 2.2. Our problem is formulated as follows: Let $\mathfrak{O}=k[[t]]$ be a formal power series ring in one variable over an algebraically closed field of characteristic zero and let $R:=\mathfrak{O}[x, y]$ be a polynomial ring in two variables over $\mathfrak{O}$. Let A be an $\mathfrak{O}$-subalgebra of R. We say that A is *cofinite* if R is a finite A-module and that A is *geometrically* $\mathfrak{O}$*-normal* if $A_K:=A\otimes_{\mathfrak{O}} K$ and $A_k:=A/tA$ are normal domains, where K is the quotient field $Q(\mathfrak{O})$ of $\mathfrak{O}$. If A is a cofinite, geometrically $\mathfrak{O}$-normal subalgebra of R, then A_K and A_k are cofinite normal subalgebras in R_K and R_k, respectively. Let $\bar{K}$ be an algebraic closure of K. We ask whether or not certain properties of a cofinite normal subalgebra $A_{\bar{K}}$ of $R_{\bar{K}}$ are inherited by the cofinite normal subalgebra A_k of R_k. We pose the following

Conjecture 1. *Let $\mathfrak{O}$ and R be as above, and let A be a cofinite, geometrically $\mathfrak{O}$-normal subalgebra of R. Then there exist a cofinite $\mathfrak{O}$-subalgebra R' of R and a finite group G of $\mathfrak{O}$-automorphisms of R' such that:*

(i) *R' is a polynomial ring in two variables over $\mathfrak{O}$ and contains A as an $\mathfrak{O}$-subalgebra;*