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ON INDECOMPOSABLE MODULES AND BLOCKS

Dedicated to Professor HIROSI NAGAO for his 60th birthday

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Introduction

Let G be a finite group and F a field of prime characteristic p. Let M be an irreducible FG-module belonging to a block B of FG with defect group D. Then the following fact is well-known. Namely if M has height 0 in B, then D is a vertex of M and the dimension of D-source of M is prime to p (provided that F is sufficiently large). The main objective of this paper is to study an indecomposable module M which satisfies the conclusion in the above statement. In particular it will turn out that M_H has a component with the same property for $H \leq G$ under certain circumstances (see Theorem 2.1). We shall apply our results to give new proofs to some of important theorems concerning blocks.

The notation is almost standard: We fix a complete discrete valuation ring R of characteristic 0 with F as its residue class field. We assume that the quotient field of R is a splitting one for every subgroup of G. We let θ denote R or F. By an θG-module M, we understand a right θG-module which is finitely generated free over θ. If M is indecomposable, we denote its vertex by $vx(M)$. For another module N, $N|M$ indicates that N is isomorphic to a direct summand of M and we say "N is a component of M" if N is indecomposable. If n is an integer and p^m is the highest p-power dividing n, then we write $m=\nu(n)$. Finally for a block B of G, we denote by $\delta(B)$ a defect group of B.

1. Sources with θ-rank prime to p

For later convenience, we put down the following well-known fact without proof.

Lemma 1.1. *Let M be an indecomposable θG-module with vertex Q. Let V be an indecomposable θQ-module. Then V is a Q-source of M if and only if $V | M_Q$ and Q is a vertex of V.*