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ON THE CONTINUITY OF PLURISUBHARMONIC FUNCTIONS ALONG CONFORMAL DIFFUSIONS

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1. Introduction

A stochastic process $Z_t=(Z_t^1, \cdots, Z_t^n)$ taking values in C^n is called a *conformal martingale* if Z_t^α and $Z_t^\alpha Z_t^\beta$, $1\leqq\alpha$, $\beta\leqq n$, are continuous local martingales. When Z_t is defined only on a time interval $[0, \eta)$ for some predictable stopping time η, Z_t is said to be a conformal martingale if so is the stopped process $Z_{t\wedge\eta'}$ for any stopping time η' strictly less than η.

Let M be a complex manifold of complex dimension n. By a diffusion process $\boldsymbol{D}=(Z_t, P_z)$ on M, we mean a strong Markov process on M with continuous sample paths defined on $[0, \zeta)$, ζ being the life time. In this paper, we assume without specific mention that the diffusion $\boldsymbol{D}$ admits no killing inside M in the sense that $P_z(\tau_U<\zeta<+\infty)=P_z(\zeta<+\infty)$, $z\in U$, for any relatively compact open set $U\subset M$, where τ_U denotes the first exit time from U: $\tau_U=\inf\{t\geqq 0: Z_t\notin U\}$. We see then that, for any open set $U\subset M$, τ_U is a predictable stopping time with respect to P_z for $z\in U$.

We call a diffusion process $\boldsymbol{D}=(Z_t, P_z)$ on M a *conformal diffusion* on M if, for any holomorphic coordinate neighbourhood (U, ϕ), the C^n-valued process $\phi(Z_t)$ defined on $[0, \tau_U)$ is a conformal martingale with respect to P_z for each $z\in U$. We occasionally assume that the transition function p_t of $\boldsymbol{D}$ is absolutely continuous with respect to a volume element V on M:

$$p_t(z, \cdot)<V\,, \qquad z\in M\,. \tag{1.1}$$

We aim at proving the following theorem.

Theorem. *Let* $\boldsymbol{D}=(Z_t, P_z)$ *be a conformal diffusion on* M *satisfying the condition* (1.1). *Then, for any plurisubharmonic function* u *on* M,

$P_z(u(Z_t)$ *is continuous in* $t\in[0, \zeta)$ *and finite for* $t\in(0, \zeta))=1$, $z\in M$.

This is a generalization of a theorem of Doob [2] to the cases of higher complex dimension and our proof is also similar to the one given in [2] in the sense that we utilize the quasi-continuity of plurisubharmonic functions with respect to a specific capacity related to the extremal function.