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A "NON-STANDARD" APPROACH TO THE FIELD EQUATIONS IN THE FUNCTIONAL FORM

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1. Nonlinear functional equations

Schwinger [1] gave a couple of equations for the Green functions of the nucleon and meson fields with interaction, in the form of functional differential equations in terms of an ordinary external field. For neutral pseudoscalar mesons, they are

$$(1.1)\quad [-i\gamma_\mu\frac{\partial}{\partial x_\mu}+m-g\gamma_5\phi(x)+ig\gamma_5\int d\xi\Delta(x,\xi;\phi)\frac{\delta}{\delta\phi(\xi)}]G(x,x';\phi)=\delta(x-x'),$$

$$(1.2)\quad (-\frac{\partial^2}{\partial\xi^2}+k^2)\Delta(\xi,\xi';\phi)=\delta(\xi-\xi')+igT_r\gamma_5\int d\eta\Delta(\xi,\eta;\phi)\frac{\delta G(\xi',\xi';\phi)}{\delta\phi(\eta)} -gT_r\gamma_5\phi(\xi)G(\xi',\xi';\phi).$$

Equations (1.1) and (1.2) form a system from which the Green functions G and Δ may be determined as functionals of the external field $\phi(x)$. By using these equations it is possible to obtain closed expressions for the complete Green functions. In electrodynamics these equations are usually treated by expanding in powers of the coupling constant. But in meson theory, this procedure is inapplicable. Edwards and Peierls [2] simplified equation (1.2) to a linear one by assuming Δ to be a given function of x and x' independent of ϕ. Starting from these equations, they obtained the Green function in an explicit form by using Fourier transform of functionals. They used a sort of functional integrals but they did not give mathematical definitions of integrations or measures with respect to which integrations are performed.

Paul Lévy [3] has developed a potential theory on an infinite dimensional space. He used "la méthode du passage du fini à l'infini" in which an infinite dimensional Laplacian and harmonic functions are defined by a limiting procedure of finite dimensional potential theory.

Recently, Hasegawa [4], [5] introduced the following space of sequences

$$(1.3)\qquad E=\{x=(x_1,x_2,\cdots,x_N,\cdots)\in R^\infty,\ \sup_N\frac{1}{N}\sum_{n=1}^N x_n^2<\infty\}$$