Maruo, K.
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ON CERTAIN NONLINEAR DIFFERENTIAL EQUATIONS OF SECOND ORDER IN TIME

KENJI MARUO

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0. Introduction

Let H be a real Hilbert space and ψ a lower semicontinuous convex proper function from H to $(-\infty, \infty]$. Here the terminology "proper" means that $\psi \not\equiv \infty$. The subdifferential of ψ is defined as follows: For $x \in H$, the value $\partial\psi x$ is the set of all $z \in H$ such that

$$\psi(y)-\psi(x) \geqq (z, y-x) \qquad \text{for every} \quad y \in H$$

where (,) stands for the inner product of H.

H. Brezis in [1] and [2] proposed the initial value problem of the form

$$(0.1) \qquad \begin{cases} \dfrac{d^2}{dt^2}u+\partial\psi u \ni f \\ u(0)=a\,, \quad \dfrac{d}{dt}u(0)=b\,. \end{cases}$$

In [1] he stated that in the particuler case where $\psi=I_K$ is the indicator function of a closed convex set K, the solution u represents, roughly speaking, the trajectory of an optical ray caught in K and reflecting at the boundary of K. Then $-\partial\psi u=-\partial I_K u$ may be regarded as the repulsive power at the boundary of K. In case H is finite dimensional, M. Schatzman made a deep investigation on this problem in [3] and [5] and established a general existence theorem as well as various results on the uniqueness and non-uniqueness of solutions. By a simple example in which ψ is the indicator function a closed convex set K she showed that the uniqueness of the solution does not hold in general and the solution which reflects optically on the boundary of K is an energy conserving solution. Moreover she obtained that even the energy conserving solutoin is not necessarily unique.

In case H is infinite dimensional, to the author's best knowledge, it seems to be extremely difficult to solve this porblem in a general situation. Hence as the first step of the study of this problem we are concerned with the case where the subdifferential operator $\partial\psi$ is expressed as