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TANGENTIAL REPRESENTATIONS OF CYCLIC GROUP ACTIONS ON HOMOTOPY COMPLEX PROJECTIVE SPACES

Dedicated to Professor Minoru Nakaoka on his sixtieth birthday

MIKIYA MASUDA[1) AND YUH-DONG TSAI

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0. Introduction

Let G be a cyclic group of an odd prime order m and let t be a generator of the complex representation ring $R(G)$ of G; i.e. $R(G)=Z[t]/(1-t^m)$. Let X be a closed G manifold homotopy equivalent to $P(C^n)$ the space consisting of complex lines in C^n. Suppose G acts smoothly on X with isolated fixed points $\{p_i\}_{i=1}^n$ (Bredon's theorem asserts the number of fixed points equals n [2]). Then the tangential representation $T_{p_i}X$ of G at p_i defines a function $\psi_i(t)$ on $G-1$ (up to multiplication by t^k) for each i; see p. 137 in [12]. In particular, if X is G homotopy equivalent to $P(A)$ for some complex representation A of G (we call such X a G homotopy $P(A)$), then it has an expression

$$\psi_i(t)=\lambda_{-1}(T_{p_i}P(A))/\lambda_{-1}(T_{p_i}X)$$

where $\lambda_{-1}(V)$ is the Euler class of a G representation V. Therefore one can regard $\psi_i(t)$ as quantities which describe to what extent the action resembles a linear action on $P(C^n)$.

Petrie's conjecture in [12] suggests that $\psi_i(t)$ is independent of i for an S^1 manifold homotopy equivalent to $P(C^n)$ and the Pontrjagin classes are preserved under the homotopy equivalence. Particularly, in [13] Petrie showed that if X is S^1 homotopy equivalent to $P(A)$ for some complex representation A of S^1, then $\psi_i(t)=\pm 1$ for each i. In contrast to S^1 actions we construct infinitely many families of G homotopy $P(C^{2d})$ such that $\psi_i(t)\neq\pm\psi_j(t)$ for $i\neq j$. Here is a brief statement of our main theorem (Theorem 4.1).

Main Theorem. *Let m be an odd prime number and $2d\,|\,m-1$ for some integer $d\geq 2$. Then there are infinitely many homotopy complex projective spaces $\boldsymbol{P}^{2d-1}$ of dimension $4d-2$ such that Z_m acts on $\boldsymbol{P}^{2d-1}$ with $2d$ isolated fixed points.*

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