Nagasaki, I.
Osaka J. Math.
22 (1985), 895–905

HOMOTOPY REPRESENTATIONS AND SPHERES OF REPRESENTATIONS

IKUMITSU NAGASAKI

(Received November 6, 1984)

0. Introduction

T. tom Dieck and T. Petrie have introduced and studied homotopy representations in [7] and [8]. Let G be a finite group. A G-CW-complex X is called a homotopy representation of G if the H-fixed point set X^H is homotopy equivalent to a (dim X^H)-dimensional sphere for each subgroup H of G. (If X^H is empty, then we set $\dim X^H=-1$ and S^{-1} is empty.) A homotopy representation of G is called linear if it is G-homotopy equivalent to a unit sphere of a real representation of G. (See [7].) We denote the set of G-homotopy classes of homotopy representations [resp. linear homotopy representations] of G by $V^+(G, h^\infty)$ [resp. $V^+(G, \ell)$]. These sets are commutative semi-groups with addition induced by join. Let $V(G, \lambda)$ be the Grothendieck group associated to $V^+(G, \lambda)$ for $\lambda=\ell$ or h^∞. We call $V(G, h^\infty)$ the homotopy representation group of G and $V(G, \ell)$ the linear homotopy representation group of G. The group $V(G, h^\infty)$ has been studied by tom Dieck and Petrie ([7], [8]) and the group $V(G, \ell)$ has been studied by many authors. (See [1], [4], [10], [11], [13], [15], [18] and [19].)

Let $\phi(G)$ be the set of conjugacy classes of subgroups of G and $C(G)$ be the ring of all integer valued functions on $\phi(G)$. For any homotopy representation X, $\mathrm{Dim}\, X\in C(G)$ is defined by $(\mathrm{Dim}\, X)\,(H)=\dim X^H+1$, which is called the dimension function of X. Since $\mathrm{Dim}\, X*Y=\mathrm{Dim}\, X+\mathrm{Dim}\, Y$ ($*$ means the join), the homomorphism $\mathrm{Dim}\colon V(G, \lambda)\to C(G)$ is induced by the assignment $X\mapsto \mathrm{Dim}\, X$. This homomorphism is called the dimension homomorphism of $V(G, \lambda)$. The kernel of Dim is denoted by $v(G, \lambda)$. tom Dieck and Petrie have shown that $v(G, h^\infty)$ is isomorphic to the Picard group $\mathrm{Pic}(A(G))$ of the Burnside ring of G in [7].

We are interested in the difference between $V(G, \ell)$ and $V(G, h^\infty)$. We observe the homomorphisms which are induced from the inclusion $V^+(G, \ell)\to V^+(G, h^\infty)$:

$$I_G\colon V(G, \ell)\to V(G, h^\infty)$$
$$i_G\colon v(G, \ell)\to v(G, h^\infty)\,.$$