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ACYCLICITY OF BP-RELATED HOMOLOGIES AND COHOMOLOGIES

Dedicated to Professor Itiro Tamura on his sixtieth birthday

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Introduction

BP is the Brown-Peterson spectrum at a fixed prime p. This spectrum is an associative and commutative ring spectrum whose homotopy is $BP_*=Z_{(p)}[v_1, \cdots, v_n, \cdots]$. For each $n \geqq 0$ there are associative BP-module spectra $P(n)$, $BP\langle n\rangle$, $k(n)$, L_nBP, M_nBP and N_nBP. If E is an associative BP-module spectrum, then we can form a weak associative BP-module spectrum $v_n^{-1}E$. When $E=P(n)$, $BP\langle n\rangle$ or $k(n)$, $v_n^{-1}E$ is written $B(n)$, $E(n)$ or $K(n)$ respectively.

For a CW-spectrum E we denote by $\langle E\rangle$ the Bousfield class of E [3]. Thus it is the equivalence class under the equivalence relation: $E\sim F$ when $E_*X=0$ if and only if $F_*X=0$. In [13] and [14] Ravenel has studied the Bousfield classes of the above BP-related spectra.

Theorem 0.1 ([13, Theorem 2.1] and [14, Lemma 3.1]).
i) $\langle B(n)\rangle=\langle K(n)\rangle=\langle M_nBP\rangle$,
ii) $\langle v_n^{-1}BP\rangle=\langle E(n)\rangle=\vee_{0\leq i\leq n}\langle K(i)\rangle=\langle L_nBP\rangle$,
iii) $\langle P(n)\rangle=\langle K(n)\rangle\vee\langle P(n+1)\rangle=\langle N_nBP\rangle$,
iv) $\langle k(n)\rangle=\langle K(n)\rangle\vee\langle HZ/p\rangle$, *and*
v) $\langle BP\langle n\rangle\rangle=\langle E(n)\rangle\vee\langle HZ/p\rangle$.

For a CW-spectrum E we denote by $\langle E\rangle^*$ the cohomological Bousfield class of E. Thus $\langle E\rangle^*=\langle F\rangle^*$ when $E^*X=0$ if and only if $F^*X=0$. Given a p-local CW-spectrum E there exists a p-local CW-spectrum ∇E related by a universal coefficient sequence

$$0 \rightarrow \mathrm{Ext}(E_{*-1}X, Z_{(p)}) \rightarrow \nabla E^*X \rightarrow \mathrm{Hom}(E_*X, Z_{(p)}) \rightarrow 0$$

(see [5] or [16]). By using this sequence we can show that $\langle\nabla E\rangle^*=\langle E\rangle$, and moreover $\langle E\rangle^*=\langle\nabla E\rangle$ if E is of finite type. The BP-module spectrum $P(n)$, $BP\langle n\rangle$, $k(n)$ or $K(n)$ is of finite type, but $v_n^{-1}BP$, $B(n)$, $E(n)$, L_nBP, M_nBP or N_nBP is not of finite type. Nevertheless we obtain

Theorem 0.2. i) $\langle B(n)\rangle^*=\langle\nabla B(n)\rangle=\langle K(n)\rangle^*=\langle\nabla K(n)\rangle=\langle K(n)\rangle$,