

A HEEGAARD-DIAGRAM OF THE 3-SPHERE

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1. Introduction

As an algorithm for recognizing S^3 in 3-manifolds, there exists the Whitehead algorithm. This algorithm is partly true by Homma-Ochiai-Takahashi [4], but in general not true by Viro [8], Ochiai [7], and Morikawa [6]. That is, Whitehead conjecture [9], [1], which asserts that all Heegaard diagram, other than the canonical one, have always waves, is true in the case of genus two, but not true in the case of genus greater than two. All already known counterexamples to the conjecture was constructed as Heegaard diagrams of 2-fold branched coverings branched along knot diagrams of the trivial knot. In this paper, we construct such a counterexample through the different method using presentations of the trivial group, and the resulting diagram has two interesting properties different from already known examples.

2. A new counterexample to Whitehead conjecture

For all definitions of Heegaard diagrams, complete systems of meridians, band moves, waves, and others we refer to [4].

At first, let's choose a trivial group

$$G = \{X, Y, Z; XY^2X^{-1} = Y^3, YX^2Y^{-1} = X^3, Z = 1\}$$

It will be noticed that the group G is trivial by Crowell-Fox [3] and by Birman-Hilden [2] there are no Heegaard diagrams which have relators of G as that of the fundamental groups induced by them. Hence we may change the relators of G and get a new presentation of the trivial group

$$\begin{aligned} H = \{X, Y, Z; & XY^2X^{-1}Y^{-1}ZY^{-1}ZY^{-1}Z = 1, \\ & YX^2Y^{-1}X^{-1}Z^{-1}X^{-1}Z^{-1}X^{-1}Z^{-1} = 1, \\ & ZXYX^{-1}Y^{-1} = 1\} \end{aligned}$$

It will be noticed that H is obtained from G by the trial and error method and that the relators of H is induced by some Heegaard diagrams of S^3 . Next