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THE CUT LOCUS AND THE DIASTASIS OF A HERMITIAN SYMMETRIC SPACE OF COMPACT TYPE

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1. Introduction

For a complete Riemannian manifold M and a point p in M, we denote by $C_p(M)$ the cut locus of M with respect to p. As a property of the cut locus of a simply connected compact symmetric space M, it is known in [2] that the cut locus $C_p(M)$ coincides with the first conjugate locus of M with respect to p. Sakai [5] proved that in general the cut locus of a compact symmetric space is determined by that of its maximal totally geodesic flat submanifold (see Section 4 for details). Using this, Sakai [6] and Takeuchi [8], [9] gave stratifications of the cut loci of compact symmetric spaces.

Calabi [1] introduced the notion "diastasis" to study Kähler imbeddings. The diastasis of a Kähler manifold M is a real analytic function defined on its domain of real analyticity in $M\times M$ containing the diagonal set and behaves like as the square of the geodesic distance in the small (see Section 2 for the definition). The most characteristic property of the diastasis proved by Calabi will be that the diastasis of a Kähler submanifold N of a Kähler manifold M coincides with the restriction of the diastasis of M to N. Making use of these properties, Calabi obtained various fundamental results of Kähler imbeddings. In particular, he proved the rigidity of a Kähler submanifold of a space of constant holomorphic sectional curvature.

It seems to be interesting to study relations between the geodesic distance and the diastasis in the large. In this note we shall show a relation between the cut locus and the diastasis of a Hermitian symmetric space of compact type. More precisely, the main result of this note is the following:

Theorem. *Let M be a Hermitian symmetric space of compact type and D be the diastasis of M. Then, for each point p in M, the cut locus $C_p(M)$ is equal to the set of points q at which $D(p, q)$ cannot be defined.*

In other words, $M-C_p(M)$ is the domain of real analyticity of the real analytic function $q\mapsto D(p, q)$. This result gives a relation between the cut locus of a Hermitian symmetric space of compact type and that of its symmetric