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LINEARLY COMPACT MODULES OVER HNP RINGS

Dedicated to Professor Hirosi Nagao for his 60th birthday

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Let R be a hereditary noetherian prime ring (an HNP ring for short) and let F be a non-trivial right Gabriel topology on R, i.e., F consists of essential right ideals of R (see §1 of [9]). Then R is a topological ring with elements of F as a fundamental system of neighborhoods of 0. Let M be a topological right R-module with a fundamental system of neighborhoods of 0 consisting of submodules. Then M is called *F-linearly compact* (*F-l.c.* for short) if
(i) it is Hausdorff,
(ii) if every finite subset of the set of congruences $x\equiv m_\alpha \pmod{N_\alpha}$, where N_α are closed submodules of M, has a solution in M, then the entire set of the congruences has a solution in M.

This paper is concerned with *F-l.c.* modules over HNP rings in the case F is special. Let A be a maximal invertible ideal of R and let F_A be the right Gabriel topology consisting of all right ideals containing some power of A. Then we give, in §2, a complete algebraic structure of F_A-*l.c.* modules by using Kaplansky's duality theorem and basic submodules. From this result we get: "F_A-*l.c.* modules"$\Rightarrow$"F_A^ω-pure injective modules". This implication is not necessary to hold for any right Gabriel topology as it is shown in §3. It is established that there is a duality between F_A-*l.c.* modules and left $\hat{R}_A$-modules, where $\hat{R}_A$ is the completion of R with respect to A (see Theorem 2.6). Main results in this paper were announced without proofs in [11].

Concerning our terminologies and notations we refer to [8] and [9].

1. Throughout this paper, R denotes an HNP ring with quotient ring Q and $K=Q/R\neq 0$. Let F be any non-trivial right Gabriel topology on R; "*trivial*" means that either all modules are F-torsion-free or all modules are F-torsion. Then F consists of essential right ideals of R (see [9, p. 96]). Let I be any essential right ideal of R. Define $(R\colon I)_l=\{q\in Q\mid qI\subseteq R\}$. Similarly $(R\colon J)_r=\{q\in Q\mid Jq\subseteq R\}$ for any essential left ideal J of R. An ideal X of R is called *invertible* if $(R\colon X)_l X=R=X(R\colon X)_r$. In this case we have $(R\colon X)_l=(R\colon X)_r$, denoted by X^{-1}. For any right Gabriel topology F, put $Q_F=\cup (R\colon I)_l$ $(I\in F)$, *the ring of quotients of R with respect to F*. The family F_l of