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PSEUDO-RANK FUNCTIONS ON CROSSED PRODUCTS OF FINITE GROUPS OVER REGULAR RINGS

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Let R be a regular ring with a pseudo-rank function. The collection of all pseudo-rank functions of R (See [2, Ch. 17]) is denoted by $P(R)$ which is a compact convex set, and the extreme boundary of $P(R)$ is denoted by $\partial_e P(R)$. Our main objective is to study a crossed product R^*G of a finite multiplicative group G over a regular ring R. A crossed product R^*G of G over R is an associative ring which is a free left R-module containing an element $\bar{x} \in R^*G$ for each $x \in G$ and the set generated by the symbols $\{\bar{x}\colon x \in G\}$ is a basis of R^*G as a left R-module. Hence every element $\alpha \in R^*G$ can be uniquely written as a sum $\alpha = \sum_{x\in G} r_x \bar{x}$ with $r_x \in R$. The addition in R^*G is the obvious one and the multiplication is given by the formulas

$$\bar{x}\bar{y} = t(x, y)\overline{xy} \quad r\bar{x} = \bar{x}r^{\tilde{x}}$$

for all $x, y \in G$ and $r \in R$. Here the twisting $t\colon G\times G \to U(R)$ is a map from $G\times G$ to the group of units of R and for fixed $x \in G$, the map $\tilde{x}\colon r \to r^{\tilde{x}}$ is an automorphism of R. We assume throughout this note that the order $|G|$ of G is invertible in R. The Lemma 1.1 of [9] implies that R^*G is also a regular ring. First we will study the question whether a pseudo-rank function P of R can be extended to one of R^*G. We shall show that P is extensible to R^*G if and only if P is G-invariant, i.e., $P(r) = P(r^{\tilde{x}})$ for all $r \in R$ and $x \in G$. More precisely for a G-invariant pseudo-rank function P, put $P^G(\alpha) = |G|^{-1}\sum_1^n P(r_i)$ for $\alpha \in R^*G$ if ${}_R(R^*G\alpha) \cong \bigoplus_1^n Rr_i$, where $r_i \in R$. Then P^G is a desired one of P.

R admits a pseudo-metric topology induced by each $P \in P(R)$. In [2, Ch. 19], K.R. Goodearl has studied the structure of the completion of R with respect to P-metric. Let $\bar{R}$ be the P-completion of R, let $\bar{P}$ be the extension of P on $\bar{R}$ and let $\phi\colon R \to \bar{R}$ be the natural ring map, Our theorems are following:

(1) There exists a crossed product $\bar{R}^*G$ and a ring map $\bar{\phi}\colon R^*G \to \bar{R}^*G$ such that the following diagram commute