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FINITE DIRECT SUM OF UNIFORM MODULES

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In a paper of M. Harada [3], a right Artinian serial (resp. coserial) ring is characterized as a right *QF*-2 (resp. *QF*-2*) ring satisfying that the class of all finite direct sums of hollow (resp. uniform) modules is closed under submodules (resp. factor modules). In his another paper [1], a new class of right Artinian rings satisfying the above condition and that any hollow module is quasi-projective is determined as a generalization of right serial rings. The main purpose of this paper is to give a generalization of right coserial rings in dual manner.

In this paper, R denotes a right Artinian ring with identity element and every module is a unitary right R-module, unless otherwise stated. For a module M, we denote its socle and injective hull as $\mathrm{Soc}(M)$ and $E(M)$, respectively, and put $S_0(M)=0$ and $S_n(M)/S_{n-1}(M)=\mathrm{Soc}(M/S_{n-1}M))$, inductively. We denote a direct sum of k-copies of M as $M^{(k)}$.

Let U and V be uniform modules of finite length with $\mathrm{Soc}(U)\simeq\mathrm{Soc}(V)$, and set $S=\mathrm{Soc}(U)$ and $E=E(U)$, then we may assume that V is a submodule of E. We shall write Δ for $\mathrm{End}_R(S)$. We can obtain the mapping φ from $\mathrm{End}_R(E)$ to Δ by the restriction to S. Since E is injective, φ is an epimorphism. While we shall denote the image of the restriction mapping from $\mathrm{Hom}_R(U, V)$ to Δ as $\Delta(U, V)$ and $\Delta(U)$ instead of $\Delta(U, U)$. It is known that $\Delta(U)$ is a subdivision ring of Δ, so we shall denote the left dimension of Δ over $\Delta(U)$ as $\dim U$, if it is finite.

A right coserial ring R satisfies the following conditions:

d–I: Every factor module of any direct sum of uniform modules of finite length is also a direct sum of uniform modules.

d-II: Every uniform module is quasi-injective.

Our purpose is to determine rings which satisfy the above both conditions, that is, we shall give the following theorem:

Theorem 1 [cf. 1: Theorem 2]. *For a right Artinian ring R, the following statements are equivalent*:

(1) *R satisfies the conditions* d-I *and* d-II.

(2) *R satisfies the condition* d-I *for direct sum of three uniform modules, and the condition* d-II.