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ON NON-SINGULAR FPF-RINGS II

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In [2], we have proved that a right non-singular ring R is right FPF ($=$every finitely generated faithful right R-module generates the category of right R-modules) if and only if (1) R is right bounded, (2) The multiplication map $Q\otimes_R Q\rightarrow Q$ is an isomorphism and Q is flat as a right R-module, where Q means the maximal right quotient ring of R, (3) For any finitely generated right ideal I of R, $Tr_R(I)\oplus r_R(I)=R$ (as ideals), where $Tr_R(I)$ means the trace ideal of I and $r_R(I)$ means the right annihilator ideal of I. This characterization implies a following result of S. Page. "Let R be a right non-singular right FPF-ring and Q be the maximal right quotient ring of R. Then Q is also right FPF and is isomorphic to a finite direct product of full matrix rings over abelian regular self-injective rings." However, as we can see from an example in section 1, not all non-singular right FPF-rings arise in this fashion.

Therefore, in this paper, we shall give a necessary and sufficient condition for a non-singular right FPF-ring to split into a finite direct product of full matrix rings over FPF-rings whose maximal right quotient rings are abelian regular self-injective rings. More precisely, we shall prove the following theorem.

Theorem 1. *Let R be a non-singular right FPF-ring. Then the following conditions are equivalent.*

(1) $R\simeq\prod_{i=1}^{t} M_{n(i)}(S_i)$, *where each S_i is a non-singular right FPF-ring whose maximal right quotient ring is an abelian regular self-injective ring.*

(2) *R contains a faithful and reduced FPF idempotent and R satisfies general comparability.*

By Y. Utumi [5], non-singular (right) continuous rings are shown to be (Von Neumann) regular, and S. Page has determined the structure of regular (right) FPF-rings. Therefore we are interested in the structure on non-singular right quasi-continuous, right FPF-rings. In section 2, as an application of Theorem 1, we shall determine the structure of non-singular right quasi-continuons right FPF-rings.