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ON NON-SINGULAR FPF-RINGS I

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A ring R is right finitely pseudo Frobenius (FPF) if every finitely generated faithful right R-module generates the category of right R-modules. In [2], C. Faith has shown that a commutative ring R is FPF if and only if (1) The total quotient ring K of R is injective, and (2) Every finitely generated faithful ideal is projective. In particular, as in case that R is a commutative semiprime ring, he has also shown that R is FPF if and only if the total quotient ring K of R is injective and R is semihereditaty.

On the other hand, S. Page [8] has proved that a (Von Neumann) regular ring R is (right) FPF if and only if R is isomorphic to a finite direct product of full matrix rings over abelian regular self-injective rings. Therefore we shall require a characterization of arbitrary FPF-rings, which involves above results.

In this paper, we shall concerned with non-singular rings. In section 1, we shall give a characterization of non-singular (resp. semihereditary) FPF-rings, which involves the theorems of C. Faith and S. Page. Further we shall give another characterization of commutative semiprime FPF-rings. In section 2, we shall present some examples.

0. Preliminaries

Throughout this paper, we assume that a ring R has identity and all modules are unitary.

Let R be a ring and M (resp. N) be a right (resp. left) R-module. Then we use $r_R(M)$ (resp. $l_R(N)$) to denote the right (resp. left) annihilator ideal of M (resp. N), and we use $Tr_R(M)$ to denote the trace ideal of M, i.e. $Tr_R(M)=\sum_{f\in M^*} f(M)$, where M^* means that the dual module of M. Further we use $Z_r(M)$ to denote the singular submodule of M, and $L_r(M)$ (resp. $L_l(N)$) to denote the lattice of right (resp. left) R-submodules of M (resp. N).

For any right R-module M, M is said to have the extending property of modules for $L_r(M)$ if for any A in $L_r(M)$, there exists a direct summand A^* of M such that $A\subseteq_e A^*$, where the notation $A\subseteq_e A^*$ means that A is an essential submodule of A^*.

For any ring R, we use $B(R)$ to denote the set of all central idempotents