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MULTIPLY TRANSITIVITY OF PERFECT 1-CODES IN SYMMETRIC GROUPS

Dedicated to Professor Hirosi Nagao on his 60th birthday

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1. Introduction

Let n be a positive integer, S_n be the symmetric group on $X=\{1, \cdots, n\}$, T be the set of all transpositions in S_n and $U=T\cup\{1\}$. A subset Z of S_n is a 1-*code* in S_n if $Ug\cap Uh=\phi$ holds for any distinct two elements g and h in Z. A 1-code Z in S_n is *perfect* if $S_n=\bigcup_{g\in Z} Ug$ (see [1]). Let $X^{(k)}$ be the set of all ordered k-tuples of distinct elements of X. We consider the natural action of S_n on $X^{(k)}$. A subset Z of S_n is k-transitive if the following condition holds.

For any x and y in $X^{(k)}$, there exists some z in Z that moves x to y, and the number of such elements in Z is a constant that is independent of the choice of x and y.

In this paper we shall prove the following result.

Theorem. *Perfect* 1-*codes in symmetric group of degree n are k-transitive for* $0\leqq k<(n/2)$.

From the above theorem we easily get the following corollary by counting the number of elements of Z that move x to y for fixed $x, y\in X^{(k)}$.

Corollary. *If* S_n *has a perfect* 1-*code then* $\binom{n}{2}+1$ *divides* $\lfloor(n/2)+1\rfloor!$.

In [4] Rothaus and Thompson proved that if S_n has a perfect 1-code then $\binom{n}{2}+1$ is not divisible by any prime exceeding $\sqrt{n}+1$. Their proof is based on the theory of group characters. In this paper we will give a combinatorial proof without using group characters.

Throughout this paper we assume that n is a fixed positive integer and S_n has a perfect 1-code Z. We shall use the following notations.

NOTATIONS

$X=\{1, \cdots, n\}$.