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ON THE UNIQUENESS OF MARKOVIAN SELF-ADJOINT EXTENSION OF DIFFUSION OPERATORS ON INFINITE DIMENSIONAL SPACES

MASAYOSHI TAKEDA

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1. Introduction

Let $(\mathcal{S}(R^d), L^2(R^d), \mathcal{S}'(R^d))$ be a rigged Hilbert space, where $\mathcal{S}(R^d)$ is the Schwartz space of test functions and $\mathcal{S}'(R^d)$ is its dual space. Letting $\{e_i\}_{i=1}^\infty \subset \mathcal{S}(R^d)$ be a complete orthonormal basis of $L_2(R^d)$, we put $FC_0^\infty=\{f; f$ is a function on $\mathcal{S}'(R^d)$ of the form $f(\xi)=\tilde{f}(\langle\xi, e_{i_1}\rangle, \cdots, \langle\xi, e_{i_n}\rangle)$ for some n and a real $C_0^\infty(R^n)$-function $\tilde{f}\}$, where $\langle\ ,\ \rangle$ is the dualization between $\mathcal{S}'(R^d)$ and $\mathcal{S}(R^d)$. Let ν be a quasi-invariant measure on $\mathcal{S}'(R^d)$ with respect to $\mathcal{S}(R^d)$. We call the measure ν *admissible* if the symmetric bilinear form $\mathcal{E}_\nu(u, v)=\frac{1}{2}(Du, Dv)_{L^2(R^d)\otimes L^2(\nu)}$, $u, v\in FC_0^\infty$, is closable. Its closed extension $(\mathcal{E}_\nu, \mathcal{F}_\nu)$ is said to be the *energy form* associated with the quasi-invariant admissible measure ν. Here, $Du=\sum_{i=1}^\infty e_i\otimes D_iu\in L^2(R^d)\otimes L^2(\nu)$ and D_i is a derivative in the direction of e_i. Furthermore, a self-adjoint operator H_ν representing the energy form $(\mathcal{E}_\nu, \mathcal{F}_\nu)$ is said to be a *diffusion operator*. For example, the probability measure μ_0 on $\mathcal{S}'(R^d)$ defined by the following formula is quasi-invariant and admissible:

$$\int_{\mathcal{S}'(R^d)} e^{i\langle\xi,\phi\rangle}\, d\mu_0(\xi) = e^{-1/4(\phi,(\Delta+m^2)^{-1/2}\phi)},\ \phi\in\mathcal{S}(R^d)\,,$$

where $(\ ,\ )$ is the scalar product in $L^2(R^d)$.

Let μ_0^* be the Euclidian random field $\langle\xi^*, \psi\rangle$ over R^{d+1}, defined by

$$\int_{\mathcal{S}'(R^{d+1})} e^{i\langle\xi^*,\psi\rangle}\, d\mu_0^*(\xi^*) = e^{-1/2(\psi,(\Delta+m^2)^{-1}\psi)},\ \psi\in\mathcal{S}(R^{d+1})\,.$$

The random field $\langle\xi^*, \psi\rangle$ can be regard as the restriction to $\mathcal{S}(R^{d+1})$ of the generalized random field indexed by the Sobolev space H_{-1} the completion of $\mathcal{S}(R^{d+1})$ with respect to the norm $\|(-\Delta+m^2)^{-1/2}\psi\|$. We denote by Σ_0 the σ-field generated by random variable $\{\langle\xi^*, \delta_0\otimes\phi\rangle;\ \phi\in\mathcal{S}(R^d)\}$, and regard the restriction of μ_0^* to Σ_0 as the measure on $\mathcal{S}'(R^d)$ by the natural identifica-