Maruyama, G.
Osaka J. Math.
22 (1985), 697–732

WIENER FUNCTIONALS AND PROBABILITY LIMIT THEOREMS I: THE CENTRAL LIMIT THEOREMS

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(Received November 1, 1984)

1. Introduction

Our object is to study limit theorems in relation to functionals built on a dynamical system generated by the flow of Gaussian white noise or equivalently functionals subordinate to a real Gaussian stationary process

$$\xi(t) = \int_{-\infty}^{\infty} \exp i\lambda t \, d\beta(\lambda) \,, \tag{1.1}$$

with $E\xi(t)=0$, complex spectral measure $d\beta$, and spectral measure $d\sigma(\lambda)=E|d\beta(\lambda)|^2$, which is absolutely continuous with respect to Lebesgue measure, $d\sigma(\lambda)=f(\lambda)d\lambda$.

Define $\mathcal{L}_{k,p}(1\leq k<\infty,\ 0<p\leq\infty)$ to be the set of complex symmetric Borel functions h on $\boldsymbol{R}^k$ satisfying (i) $\overline{h(\lambda)}=h(-\lambda)$ (ii) $h\in L^p(d^k\sigma)$, $d^k\sigma=d\sigma(\lambda_1)d\sigma(\lambda_2)\cdots d\sigma(\lambda_k)$, $\lambda=(\lambda_1,\cdots,\lambda_k)\in R^k$. A real-valued second order strictly stationary process $X(t)$, subordinate to ξ, with zero mean is represented by the Ito-Wiener expansion

$$X(t) = \sum_{k=1}^{\infty} X_k(t) \,, \quad X_k(t) = \int c_k(\lambda)e_k(\lambda,t)d^k\beta \,, \tag{1.2}$$

where $c_k\in\mathcal{L}_{k,2}$, $e_k(\lambda,t)=\exp i\bar{\lambda}t$, $\bar{\lambda}=\lambda_1+\cdots+\lambda_k$, $d^k\beta=d\beta(\lambda_1)\cdots d\beta(\lambda_k)$; the k-fold multiple Ito Integral ([2], [3]) in (1.2) is understood in the usual way ([6], [7], [8]). Throughout the paper the whole space as an integration domain is suppressed in an integral sign. $\boldsymbol{R}^k$ is such an example for the integral in (1.2).

Summarizing notational conventions: Constants will be denoted by c, c_1, c_2, $\cdots$ which are not always the same for each appearance. Given non-negative $f(x)$, $g(x)$, we use $f(x)\asymp g(x)$ to indicate that there exist constants c_1, $c_2>0$ such that $c_1f(x)\leq g(x)\leq c_2f(x)$ on a specified region.

To formulate the main theorem introduce an integral transform which maps $u\in L^1(d^k\sigma)$ $(k\geq1)$ to $\varphi(u;\lambda)\in L^1(\boldsymbol{R})$, the space of Lebesgue integrable functions on $\boldsymbol{R}$: