Azzam, A.
Osaka J. Math.
22 (1985), 691–696

ON MIXED BOUNDARY VALUE PROBLEMS FOR PARABOLIC EQUATIONS IN SINGULAR DOMAINS

ALI AZZAM

(Received February 28, 1984)

1. Introduction. In this paper we continue our investigation on boundary value problems for elliptic and parabolic equations in singular domains. The problem is thoroughly investigated if the boundary is smooth. See [1] for general boundary value problems for elliptic equations and [8] and [11] for the parabolic case.

Elliptic boundary value problems in singular domains have been studied by many authors using different approaches. See [10], [12]–[15]. Comparatively little is known in the case of parabolic equations. One of the reasons for that is the fact that the methods used in the elliptic case do not extend completely to the parabolic case. In [3] we have introduced a method for investigating the Dirichlet problem for elliptic equations in plane domains with corners. This method was then modified to study different boundary value problems for elliptic equations in n-dimensional domains with edges (cf. [4], [5]) and initial-Dirichlet problem for parabolic equations (cf. [6]). The method is based on obtaining a bound for the solution near the singular part of the boundary. This is done by constructing a suitable barrier function. Then using a Schauder-type estimate we obtain bounds for the derivatives of the solution and then its smoothness properties. In [7], we applied this method to investigate the smoothness properties of solutions of initial-mixed boundary value problems for parabolic equations and obtained C^{ν} statements for these solutions, $1<\nu\leq 2$. In this paper, we study the same problem, and give conditions sufficient for the solution to belong to $C^{m+2+\alpha}$, $m\geq 0$, $0<\alpha<1$.

2. The problem. Consider a simply connected bounded domain $G\subset R^2$ with boundary consisting of finite number of $C^{m+2+\alpha}$ curves $\Gamma_1, \cdots, \Gamma_q$. Here $m\geq 0$ is an integer and $\alpha\in(0, 1)$. Γ_k, Γ_{k+1} meet at the point $x^{(k)}=(x_1^{(k)}, x_2^{(k)})$ forming there an interior angle γ_k; $0<\gamma_k<2\pi$, $k=1, \cdots, q$, $\Gamma_{q+1}=\Gamma_1$. In $\Omega=G\times J$ where $J=\{t: 0\leq t<T\}$ consider the parabolic operator $Lu\equiv a_{ij}(x, t)u_{ij}+a_i(x, t)u_i+a(x, t)u-u_t$ Here $x=(x_1, x_2)$, $u_i=\dfrac{\partial u}{\partial x_i}$, $u_{ij}=\dfrac{\partial^2 u}{\partial x_i\partial x_j}$, $i, j=1, 2$ and we use the summation convention. Consider in Ω the initial mixed boundary value