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TRAPPING OBSTACLES WITH A SEQUENCE OF POLES OF THE SCATTERING MATRIX CONVERGING TO THE REAL AXIS

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1. Introduction. We consider the scattering of the acoustic equation by bounded obstacles. Let $\mathcal{O}$ be a bounded open set in $\boldsymbol{R}^3$ with sufficiently smooth boundary. We set $\Omega=\boldsymbol{R}^3-\bar{\mathcal{O}}$. Suppose that Ω is connected. Consider the following problem

$$\begin{cases} \Box u = \dfrac{\partial^2 u}{\partial t^2} - \sum_{j=1}^{3} \dfrac{\partial^2 u}{\partial x_j^2} = 0 & \text{in } (-\infty, \infty)\times\Omega \\ u(t, x) = 0 & \text{on } (-\infty, \infty)\times\Gamma\,. \end{cases}$$

Denote by $\mathcal{S}(z)$ the scattering matrix for this problem. About the definition and the fundamental properties of the scattering matrix, see Lax and Phillips [8], especially Theorems 5.1 and 5.6 of Chapter V.

On relationships between geometric properties of $\mathcal{O}$ and the location of poles of $\mathcal{S}(z)$ Lax and Phillips gave a conjecture [8, page 158] (see also Ralston [16, 17]), that is, for a nontrapping obstacle the scattering matrix $\mathcal{S}(z)$ is free for poles in $\{z;\ \text{Im}\ z\leqslant\alpha\}$ for some constant $\alpha>0$, and for a trapping obstacle $\mathcal{S}(z)$ has a sequence of poles $\{z_j\}_{j=1}^{\infty}$ such that $\text{Im}z_j\to 0$ as $j\to\infty$. Concerning this conjecture Morawetz, Ralston and Strauss [14] and Melrose [11] proved that the part for nontrapping obstacles is correct. On the other hand, Bardos, Guillot and Ralston [1], Petkov [15] and Ikawa [4, 5, 6] made considerations on some simple cases of trapping obstacles. Among them the result of Ikawa [4, 5] shows that the part of the conjecture for trapping obstacles is not correct in general, namely for two strictly convex objects $\mathcal{S}(z)$ is free for poles in $\{z;\ \text{Im}z\leqslant\alpha\}$ $(\alpha>0)$. Yet it seems very sure that the conjecture remains to be correct for a great part of trapping obstacles. In spite of the conjecture we have not known even an example of obstacle $\mathcal{O}$ for which is proved the existence of a sequence of poles of the scattering matrix converging to the real axis.[1)]

The purpose of this paper is to show an example of $\mathcal{O}$ whose scattering

1) Ralston [16] gives examples of the scattering by the inhomogeneity of medium such that the scattering matrix has a sequence of poles converging to the real axis.