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ASYMPTOTIC PROPERTY OF AN EIGENFUNCTION OF THE LAPLACIAN UNDER SINGULAR VARIATION OF DOMAINS —THE NEUMANN CONDITION—

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1. Introduction

We consider a bounded domain Ω in $\boldsymbol{R}^2$ with smooth boundary γ. Let B_ε be the ε-disk whose center is $\tilde{w}\in\Omega$. We put $\Omega_\varepsilon=\Omega\backslash\bar{B}_\varepsilon$. We consider the following eigenvalue problems (1.1) and (1.2):

$$
(1.1) \qquad \begin{aligned}
-\Delta_x u(x) &= \lambda(\varepsilon)u(x)\,, && x\in\Omega_\varepsilon\,,\\
u(x) &= 0\,, && x\in\gamma\,,\\
\frac{\partial u}{\partial \nu}(x) &= 0\,, && x\in\partial B_\varepsilon\,,
\end{aligned}
$$

where $\partial/\partial\nu$ denotes the derivative along the inner normal vector at x with respect to the domain Ω_ε.

$$
(1.2) \qquad \begin{aligned}
-\Delta_x u(x) &= \lambda u(x)\,, && x\in\Omega\,,\\
u(x) &= 0\,, && x\in\gamma\,.
\end{aligned}
$$

Let $0<\mu_1(\varepsilon)\leq\mu_2(\varepsilon)\leq\cdots$ be the eigenvalues of (1.1). Let $0<\mu_1\leq\mu_2\leq\cdots$ be the eigenvalues of (1.2). We arrange them repeatedly according to their multiplicities. Denote by $\{\varphi_j(\varepsilon)\}_{j=1}^\infty$ ($\{\varphi_j\}_{j=1}^\infty$, respectively) a complete orthonormal basis of $L^2(\Omega_\varepsilon)$ ($L^2(\Omega)$, respectively) consisting of eigenfunction of $-\Delta$ associated with $\{\mu_j(\varepsilon)\}_{j=1}^\infty$ ($\{\varphi_j\}_{j=1}^\infty$, respectively).

In this note we consider the following problem:
Problem. What can one say about asymptotic behaviour of $\varphi_j(\varepsilon)$ as ε tends to zero?

It is well known that $\mu_j(\varepsilon)$ tends to μ_j as ε tends to zero. See Rauch-Taylor [8], Ozawa [5]. As a consequence, $\mu_j(\varepsilon)$ is simple for small $\varepsilon>0$, if we assume that μ_j is simple. Thus $\varphi_j(\varepsilon)$ is uniquely determined up to the arbitratiness of multiplication by $+1$ or -1.

We have the following Theorem 1. Theorem 2 is our main result.