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THEOREMS ON LARGE DEVIATIONS FOR A FAMILY OF STOCHASTIC PROCESSES CONVERGING TO A MARKOV PROCESS

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Introduction

In their powerful consecutive works [5]–[7], [9], [12], Donsker and Varadhan have developed the theory of large deviations for Markov processes. Applying their fundamental theorems, they also obtained several remarkable results [8], [10], [11], [13] concerning the large-time asymptotics for certain Markov processes. Although each theorem in their works on general theory involves the probabilities of large deviations for a single Markov process, it is quite natural in some applications (see [11], [19], [16]) to consider those for a family of stochastic processes. In the present paper we study the theory of large deviations for a certain family of stochastic processes converging to a Markov process, which corrseponds to the general theory in [12]. As its applications we prove some theorems on the Chung type laws of iterated logarithm, generalizing some reuslts in [10].

Let X be a Polish space and let Ω [resp. Ω^+] denote the space of X-valued right-continuous functions on $(-\infty, +\infty)$ [resp. $[0, +\infty)$] without discontinuities of the second kind, endowed with the Skorohod type topology. Let θ_t denote the shift operator on Ω, i.e., $\theta_t\omega=\omega(t+\cdot)$. For any $t>0$ define $p_t\colon \Omega^+\to\Omega$ by $(p_t\omega)(s)=\omega(s)$, $0\leqq s<t$, and $(p_t\omega)(s+t)=(p_t\omega)(s)$, $-\infty<s<\infty$. We can define for any $t>0$ and $\omega\in\Omega^+$

$$(1)\qquad R_{t,\omega}(A)=R_t(\omega, A)=\frac{1}{t}\int_0^t \chi_A(\theta_s p_t\omega)\,ds,\ A\subset\Omega\,.$$

Let $\mathcal{M}_S(\Omega)$ denote the space of all probability measures Q on Ω such that $Q\circ\theta_t^{-1}=Q$, $-\infty<t<\infty$, i.e., the space of all stationary processes on X. Note that $R_t(\omega, \cdot)\in\mathcal{M}_S(\Omega)$. For any $Q\in\mathcal{M}_S(\Omega)$ we denote the one-dimensional marginal of Q by $q[Q]$.

Let $\{x(t)\}$ be a homogeneous Markov process on X. In [12] Donsker and Varadhan give the definition of the *entropy* function $H(Q)$, $Q\in\mathcal{M}_S(\Omega)$, associated

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