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REMARKS ON LINEAR VOLTERRA INTEGRAL EQUATIONS OF PARABOLIC TYPE

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The purpose of this paper is to improve the result of the previous paper [6] on the linear Volterra integral equation of parabolic type

$$u(t)+\int_0^t b(t-s)A(s)u(s)ds=f(t)\,, \qquad 0\leqq t\leqq T\,, \tag{0.1}$$

in a Banach space X. Here b is a given complex valued function such that $b(0)=1$, f and u are given and unknown functions with values in X respectively, and $-A(t)$ is a closed linear operator in X which generates an analytic semi-group for each t.

In [6] assuming among others that $\dot{b}$ is absolutely continuous and $\ddot{b}\in L^p(0, T)$ for some $p>1$ (cf. Friedman-Shinbrot [4]), we constructed the fundamental solution $W(t, s)$ of (0.1) which is an operator valued function defined in $0\leqq s\leqq t\leqq T$ satisfying

$$W(t, s)+\lim_{\varepsilon\downarrow 0}\int_{s+\varepsilon}^t b(t-\tau)A(\tau)W(\tau, s)d\tau = I\,, \tag{0.2}$$

$$\left\|\frac{\partial}{\partial t}W(t, s)\right\|\leqq\frac{C}{t-s},\ \ \|A(t)W(t, s)\|\leqq\frac{C}{t-s}\,. \tag{0.3}$$

Using the fundamental solution we showed the existence and uniqueness of the solution of (0.1) such that the integral in the left side of the equation exists as an improper integral:

$$\int_0^t b(t-s)A(s)u(s)ds = \lim_{\varepsilon\downarrow 0}\int_\varepsilon^t b(t-s)A(s)u(s)ds\,. \tag{0.4}$$

Recently J. Prüss [5] constructed the fundamental solution for the equation of nonconvolution type

$$\frac{d}{dt}u(t)+A(t)u(t)=\int_0^t K(t, s)A(s)u(s)ds+f(t)\,, \tag{0.5}$$

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