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RANDOM ITERATION OF ONE-DIMENSIONAL TRANSFORMATIONS

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1. Introduction

The most familiar one-dimensional dynamical system is given by

$$x_{n+1}=f(x_n) \quad \text{for} \quad n\geqq 0\,, \tag{1.1}$$

where f is a transformation from an interval into itself. Usually f in (1.1) is assumed to be of piecewise C^2. Moreover if f is uniformly expanding the asymptotic behavior of x_n is investigated in detail (see [7], [8], and [14]). But it will be more natural to consider that f may be changed for each n, by chance. For example, let S be a measurable space, let $\{f_s\}_{s\in S}$ be a family of transformations from the unit interval I into itself and let $\{X_n\}_{n=1}^{\infty}$ be a sequence of S-valued independent and identically distributed random variables on a probability space $(\Omega, \mathcal{F}, P)$. The relation between x_n and x_{n+1} is given by

$$x_{n+1}=f_{X_{n+1}(\omega)}(x_n) \quad n\geqq 0\,. \tag{1.2}$$

In this paper, we will study the asymptotic behavior of x_n given by (1.2). Following S. Kakutani [5] and T. Ohno [9], we introduce the skew product transformation T on $I\times\Omega$ satisfying

$$\mathrm{proj}_I\circ T^n(x, \omega)=f_{X_n(\omega)}f_{X_{n-1}(\omega)}\cdots f_{X_1(\omega)}\,x, \quad n\geqq 0\,, \tag{1.3}$$

and deduce our problem to the investigation of the asymptotic behavior of T.

In section 5, we introduce an expanding condition (A.1) for the random transformations f_{X_n}'s. Under this condition, similar to a single piecewise C^2 uniformly expanding transformation, we can obtain the following results:

I. Let m denote the Lebesgue measure on I. Then T has a finite number of $(m\times P)$-absolutely continuous ergodic probability measures such that any $(m\times P)$-absolutely continuous T-invariant σ-additive finite set function can be written as a linear combination of them. These ergodic measures have disjoint supports, each of which is called an ergodic component of T.

II. Each ergodic component of T can be decomposed into finitely many exact components which are permuted cyclically by T.