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# ON TRANSLATION PLANES OF ORDER $q^2$ WHICH ADMIT AN AUTOTOPISM GROUP HAVING AN ORBIT OF LENGTH $q^2-q$

Dedicated to Professor Hirosi Nagao on his 60th birthday

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## 1. Introduction

Let $\pi$ be a translation plane of order $q^2$ satisfying the following conditions:

(1.1) $\pi$ has $GF(q)$ in its kernel.

(1.2) $\pi$ admits a linear autotopism group of order $q$. (Here a linear autotopism group is a subgroup of the linear translation complement of $\pi$ which fixes at least two points on the line $l_\infty$ at infinity.)

Several classes of translation planes with these properties have been constructed in [1], [2] and [7]. Any of these planes can be coordinatized by a quasifield having a central kernel of order $q$ and satisfy the following condition:

(1.2)′ $\pi$ admits a linear autotopism group having an orbit of length $q^2-q$ on the line at infinity.

The purpose of this paper is to investigate the translation planes with the properties (1.1) and (1.2), especially with (1.1) and (1.2)′ in the latter half of the paper.

In §2 we consider the quasifields corresponding to these planes. Let $K$ be a field. Let $h(x)$ be a mapping from $K$ into $K$ and $r(y)$ and $s(y)$ mappings from $K^\sharp=K-\{0\}$ into $K$. Set $f(x, y)=-y^{-1}(x^2-r(y)x-s(y))$ and $g(x, y)=-x+r(y)$. Assume that $r(y)$, $s(y)$ and $h(x)$ satisfy the following conditions.

(1.3) $f(x, y_1)\neq f(x, y_2)$ whenever $x\in K$ and $y_1, y_2\in K^\sharp$, $y_1\neq y_2$.

(1.4) $K=f(x, K^\sharp)\cup h(x)$ (disjoint union) for any $x\in K$.

(1.5) $h(0)=h(1)=0$.

Let $\tilde{\Phi}_K$ be the set of such triples $(r, s, h)$ and put $\Phi_K=\{(r, s, h)\,|\,(r, s, h)\in\tilde{\Phi}_K, h(x)=0 \text{ for any } x\in K\}$. An element $(r, s, 0)$ of $\Phi_K$ is often written $(r, s)$ for brevity's sake.

A quasifield $Q_{(r,s,h)}((r, s, h)\in\tilde{\Phi}_K)$, which is a two dimensional left vector space over $K$ with a basis $\{1, \lambda\}$, is defined by a multiplication