Kimura, H. and Ohmori, H.
Osaka J. Math.
**22** (1985), 401-410

# A CHARACTERIZATION OF SOME PARTIAL GEOMETRIC SPACES

Dedicated to Professor Hirosi Nagao on his 60th birthday

HIROSHI KIMURA AND HIROYUKI OHMORI

(Received April 17, 1984)

## 1. Introduction

A partial geometric space $S$ of dimension $m\geqq 2$ defined in [2, 6] consists of the sets $\{A_i\}_{i=-1}^{m}$ and the set $T$ such that the following eight axioms are satisfied:

(1) $A_i\cap A_j=\phi$ whenever $i\neq j$ and $-1\leqq i, j\leqq m$.

(2) $|A_{-1}|=|A_m|=1$.

(3) $T\subset \prod_{i=-1}^{m} A_i$.

The elements of $A_i$, $-1\leqq i\leqq m$, are called $i$ elements of $S$. The elements of $T$ are called flags of $S$. There is a property called incidence which is a relation between the elements of $S$ based on the flags.

(4) For each $i$ element $x_i$ there is a flag $(t_{-1}, \cdots, t_m)\in T$ such that $x_i=t_i$, where $-1\leqq i\leqq m$.

(5) Whenever $(y_{-1}, \cdots, y_m)\in T$ and $(z_{-1}, \cdots, z_m)\in T$ and $y_k=z_k$ for some $k$, $-1\leqq k\leqq m$, then there exists a flag $(t_{-1}, \cdots, t_m)\in T$, where $t_i=y_i$ for $-1\leqq i\leqq k$, and $t_j=z_j$ for $k\leqq j\leqq m$.

(6) If $x_i\in A_i$ and $x_j\in A_j$, then $x_i$ and $x_j$ have an $l$ intersection $x_l\in A_l$ and an $s$ join $x_s\in A_s$. Here $x_i$ and $x_j$ are said to have an $l$ intersection $x_l$ ($s$ join $x_s$), where $-1\leqq l\leqq \min\{i, j\}$ ($\max\{i, j\}\leqq s\leqq m$) if and only if $x_l$ ($x_s$) is incident with $x_i$ and $x_j$ such that whenever $x_n$ is an $n$ element of $S$ for $-1\leqq n\leqq \min\{i, j\}$ ($\max\{i, j\}\leqq n\leqq m$) which is incident with $x_i$ and $x_j$, then $x_n$ is incident with $x_l$ ($x_s$) and $-1\leqq n\leqq l$ ($s\leqq n\leqq m$). By the definition, $x_i$ and $x_j$ have unique intersection and unique join.

(7) If $x_{i-1}\in A_{i-1}$ and $x_{i+1}\in A_{i+1}$ are incident, then there are $k(i)$ $i$ elements which are incident with $x_{i-1}$ and $x_{i+1}$, where $2\leqq k(i)<\infty$, for $0\leqq i\leqq m-1$. The number $k(i)$ is independent of the choice of $x_{i-1}$ and $x_{i+1}$, and depends only on $i$. $k(0), k(1), \cdots, k(m-1)$ are called the configuration parameters of $S$.

(8) Let $m\geqq 2$. If $x_i\in A_i$ and $x_{i+1}\in A_{i+1}$ have an $(i-1)$ intersection $x_{i-1}$ and an $s$ join $x_s$, where $0\leqq i\leqq m-2$ and $i+2\leqq s\leqq m$, then there are $t(i, s, k)$ $i$