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ON GENERALIZED DECOMPOSITION NUMBERS AND FONG'S REDUCTIONS

Dedicated to Professor Hirosi Nagao on his 60th birthday

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Introduction

In this paper we investigate how generalized decomposition numbers behave under Fong's reductions.

Let G be a finite group and p be a fixed prime number. If π is a p-element of G and B is a p-block of G, then for an ordinary irreducible character χ in B and for each p-regular element ρ of the centralizer $C_G(\pi)$ of π, we have

$$\chi(\pi\rho)=\sum_{\phi} d(\chi, \pi, \phi)\phi(\rho) .$$

Here ϕ ranges over the irreducible Brauer characters in the p-blocks of $C_G(\pi)$ associated with B. We have the following theorem related to the Fong's first reduction.

Theorem 1. *Let H be a subgroup of G, and let B and $\tilde{B}$ be p-blocks of G and H, respectively. We assume that $\tilde{\chi}\to\tilde{\chi}^G$ is a 1–1 correspondence between the ordinary irreducible characters in $\tilde{B}$ and those in B, where $\tilde{\chi}^G$ is the character of G induced from $\tilde{\chi}$. Then the following holds.*

(i) *B and $\tilde{B}$ have a common defect group D.*

(ii) *Let $\tilde{b}$ be a root of $\tilde{B}$ in $C_H(D)D$. Then $\tilde{b}^{C_G(D)D}$ is defined in the sense of Brauer* [2]. *We put $b=\tilde{b}^{C_G(D)D}$. Then b is a root of B in $C_G(D)D$ and $T(b)=T(\tilde{b})C_G(D)$ where $T(b)$ is the inertial group of b in $N_G(D)$ and $T(\tilde{b})$ is the inertial group of $\tilde{b}$ in $N_H(D)$. In particular $T(b)/C_G(D)D\cong T(\tilde{b})/C_H(D)D$.*

(iii) *Let $\{(\pi_i, \tilde{b}_i), i=1, 2, \cdots, n\}$ be a set of representatives for the conjugacy classes of subsections associated with $\tilde{B}$. Then $\tilde{b}_i^{C_G(\pi_i)}$ is defined and $\tilde{\phi}\to\tilde{\phi}^{C_G(\pi_i)}$ is a 1–1 correspondence between the irreducible Brauer characters in $\tilde{b}_i$ and those in $\tilde{b}_i^{C_G(\pi_i)}$. Furthermore $\{(\pi_i, \tilde{b}_i^{C_G(\pi_i)}), i=1, 2, \cdots, n\}$ is a set of representatives for the conjugacy classes of subsections associated with B.*

(iv) *Let $\tilde{\chi}$ be an ordinary irreducible character in $\tilde{B}$ and $\tilde{\phi}$ be an irreducible Brauer character in $\tilde{b}_i$. Then*