Jha, V. and Johnson, N. L.
Osaka J. Math.
22 (1985), 365-377

ON SPREADS OF CHARACTERISTIC p ADMITTING NONSOLVABLE GROUPS, WHOSE SYLOW p-SUBGROUPS ARE PLANAR

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(Received May 17, 1984)

Introduction

In this paper, π denotes a translation plane of order n and characteristic p admitting, in its translation complement, a group G such that $\sqrt{n} \mid |G|$. We use S to denote a Sylow p-subgroup of G and assume that S is planar with fixed plane π_S.

The recently completed classification due to Foulser and Johnson [9, 10], of the finite translation planes of order n that admit $SL(2, \sqrt{n})$ in their translation complement, has the following implication for π.

Theorem A. *Suppose the plane π_S is a Baer subplane of π and let $\mathcal{P}_n$ denote the class of nonisomorphic translation planes of order n that admit $SL(2, \sqrt{n})$ in their translation complement. Then both the following statements are valid.*

(a) *If $S \lhd\!\lhd G$ then π is isomorphic to a plane in $\mathcal{P}_n$.*

(b) *If G is nonsolvable then either $S \lhd\!\lhd G$, and so part* (a) *applies, or π_S is a G-invariant Desarguesian subplane such that restriction homomorphism $G \to G|\pi_S$ has solvable kernel but its image is a (meta) cyclic extension of $SL(2, 5)$.*

Proof. Suppose $S \lhd\!\lhd G$. It is now sufficient to verify that $G \supseteq SL(2, \sqrt{n})$. For odd $n(\neq 9)$ this follows from Foulser [7, corollary 4.2 and proposition 5.1], while for n even we get the required conclusion from Dempwolff [2, Satz, p. 1]. Since $n=9$ presents no problems, case (a) applies. If G is nonsolvable and $S \lhd G$ then the conclusions of case (b) are easily deduced from Foulser [6]: the details are very similar to the proof of proposition 4.4.

The object of the present paper is to attempt a generalization of theorem A by weakening the assumption imposed on π_S; specifically, instead of assuming that π_S is a Baer subplane of π, we shall assume that π_S has order at least $n^{1/4}$. If n is odd, and this is the case we shall mainly be concerned with, it turns out that the conclusions of theorem A are essentially unchanged; though there may exist a new plane of order 5^4.