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# ON ARTINIAN RINGS WHOSE INDECOMPOSABLE PROJECTIVES ARE DISTRIBUTIVE

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## 1. Introduction

A module $L \neq 0$ is called local (or hollow) if $L=L_1+L_2$ implies $L=L_1$ or $L=L_2$. Especially a noetherian module is local if and only if it has a unique maximal submodule.

A module $M$ is called distributive if $X \cap (Y+Z)=(X \cap Y)+(X \cap Z)$ for every submodules $X$, $Y$, $Z$ in $M$ (cf. [2]). It is clear that any sub- (or factor) module of a distributive module is distributive.

We call a ring $R$ right locally distributive, right $LD$ in abbreviation, if it is right artinian and every projective indecomposable right $R$-module is distributive. It is evident that every local right module over a right $LD$-ring is distributive. The class of right $LD$-rings is a generalization of the class of right serial rings.

In this note right $LD$-rings are studied, mainly to construct a number of right $LD$-algebras.

## 2. Right $LD$-rings

The following lemma, shown by Fuller, is basic to study distributive modules over a semiperfect ring.

**Lemma 1.** *Let $R$ be a semiperfect ring. The following conditions on a right $R$-module $M$ are equivalent:*

(1) *$M$ is distributive.*

(2) *For every primitive idempotent $e$ of $R$, the set $\{xeR \mid x \in M\}$ of all homomorphic images of $eR$ in $M$ is linearly ordered.*

(3) *For every primitive idempotent $e$ in $R$, the right $eRe$-module $Me$ is uniserial.*

Proof. See Fuller [1].

**Theorem 2.** *The following conditions on a right artinian ring $R$ are equivalent:*