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NOTES ON SIGNATURES ON RINGS

Dedicated to Professor Hirosi Nagao on his 60th birthday

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0. Introduction

The notion of infinite prime introduced by Harrison [3] was investigated in [1], [2], [7] and [9] which were concerned with ordering on a field. In this note, we study about signatures on rings as some generalization of infinite primes and signatures of fields in [2]. In the section 1, we introduce notions of U-prime and signature of a ring which are generalizations of infinite prime and signature of field. In the section 2, we show that a U-prime of a commuative ring defines a signature on the ring. In the sections 3 and 4, we consider the category of signatures and a space of signatures on a ring which include notions of extension of signature and space of ordering on fields (cf. [2] and [8]), and investigate them. Throughout this paper, we assume that every ring has identity 1.

1. Preliminaries, definitions and notations

Let S be a multiplicative semigroup, and T a normal subsemigroup of S, (cf. [6], p. 195), denoted by $T \lhd S$, that is, T is a subsemigroup of S which satisfies 1) for $x, y \in S$, $xy \in T$ implies $yx \in T$, 2) if there is an $x \in T$ with $xy \in T$, then $y \in T$, and 3) for every $x \in S$, there exists an $x' \in S$ with $x'x \in T$. We can define a binary relation $\sim$ on S; for $x, y \in S$, $x \sim y$ if and only if there is a $z \in S$ such that both zx and zy are contained in T. Then, the relation $\sim$ is an equivalence relation on S, and is compatible with the multiplication of S, so the quotient set $S/\sim$, denoted by S/T, makes a group such that the canonical map $\psi: S \rightarrow S/T$; $x \mapsto [x]$ is a homomorphism with Ker $\psi = T$.

Let R be any ring with identity 1, and P a preprime of R ([3]), that is, P is closed under addition and multiplication of R and $-1 \notin P$. We put $p(P) = P \cap -P$, $R_P = \{x \in R \mid xp(P) \cup p(P)x \subset p(P)\}$, $R_P^+ = R_P \backslash p(P)$ $(:= \{x \in R_P \mid x \notin p(P)\})$, $P^+ = P \backslash p(P)$ $(= P \backslash -P)$. We shall say a preprime P to be *complete quasi-prime*, if it satisfies the following conditions;

1) $p(P)$ is an ideal of R_P such that $R_P/p(P)$ is an integral domain,
2) $P^+ \lhd R_P^+$ under the multiplication of R_P.