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ON MAXIMAL SUBMODULES OF A FINITE DIRECT SUM OF HOLLOW MODULES IV

To the memory of Professor Takehiko MIYATA

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In the previous papers [1] and [2], we have studied conditions under which every maximal submodule of a finite direct sum D of certain hollow modules over a right artinian ring with 1 contains a non-zero direct summand of D. The present objective is to generalize slightly Theorems 3 and 4 of [2] related to the property mentioned above.

Throughout this paper, R will represent a right artinian ring with identity, and every R-module will be assumed to be a unitary right R-module with finite composition length. We denote the Jacobson radical and the length of a composition series of an R-module M by $J(M)$ and $|M|$, respectively. Occasionally, we write $J=J(R)$. If M has a unique maximal submodule $J(M)$, M is called hollow (local). When this is the case, $M\approx eR/A$ for some primitive idempotent e and a right ideal A in eR.

Let $\{N_i\}_{i=1}^n$ be a family of hollow modules, and $D=\sum_{i=1}^n\oplus N_i$. We are interested in the following condition [1]:

(**) *Every maximal submodule of D contains a non-zero direct summand of D.*

As was claimed in [1], [2], whenever we study the conition (**), we may restrict ourselves to the case where R is basic and $N_i=eR/A_i$ for a fixed primitive idempotent e and a right ideal A_i in eR. Now, let $N=eR/A$ be a hollow module. Put $\Delta=eRe/eJe=\overline{eRe}=\mathrm{End}_R(N/J(N))=\mathrm{End}_R(eR/eJ)$, and $\Delta(A)$ $(=\Delta(N))=\{\bar{x}\,|\,x\in eRe$ and $xA\subset A\}$ (see [2]). We denote by $N^{(m)}$ the direct sum of m copies of N. Then $N^{(m+1)}=N\oplus N^{(m)}$. If M is a maximal submodule of $N^{(m)}$ then $N\oplus M$ is a maximal submodule of $N^{(m+1)}$. Thus we get a mapping $\theta(m)$ of the isomorphism classes of maximal submodules in $N^{(m)}$ into the isomorphism classes of maximal submodules in $N^{(m+1)}$.

Theorem 1 (cf. [3], Corollary 2 to Theorem 3). *Let $N=eR/A$ be a hollow module. Then the following conditions are equivalent*:

1) $[\Delta\colon\Delta(A)]=k$.

2) *If $m>k$, every maximal submodule M in $D=N^{(m)}$ contains a submodule*