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Osaka J. Math.
22 (1985), 307–320

APPLICATIONS OF MALLIAVIN'S CALCULUS TO TIME-DEPENDENT SYSTEMS OF HEAT EQUATIONS

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(Received June 8, 1984)
(Revised September 11, 1984)

1. Introduction

Recently, various applications of Malliavin's calculus are studied by several authors. In particular, Kusuoka-Stroock applied Malliavin's calculus to the investigation of second order differential operators of Hörmander type ([5]). In fact, they have shown that the semigroup generated by a differential operator $L=\frac{1}{2}\sum_1^r(V_i)^2+V_0$, V_j's being all C_b^∞-vector fields on R^d, has a C^∞-kernel if V_j's satisfy the restricted Hörmander condition (cf. [8], [5]). Furthermore, they showed that the above L is hypoelliptic when the general Hörmander condition is satisfied ([5]). Our aim of this paper is to extend their result to a time-dependent system associated with an operator A(s): $C^\infty(R^d; R^d)\to C^\infty(R^d; R^d)$, where $C^\infty(R^d; R^d)$ is the space of all C^∞-mappings of R^d into itself. Indeed, suppose that the operator A(s) is represented as

$$(1)\qquad (A(s)f)_j(x) = ([\frac{1}{2}\sum_{i=1}^r(V_i(s))^2+V_0(s)]f_j)(x) +\sum_{i=1}^r\sum_{m=1}^d a_j^{im}(s, x)(V_i(s)f_m)(x)+\sum_{m=1}^d c_j^m(s, x)f_m(x),$$

where $f=(f_1, \cdots, f_d)\in C^\infty(R^d; R^d)$, $V_j(s)$'s are time-dependent C^∞-vector fields on R^d and $a_j^{im}(s)$, $c_j^m(s)\in C^\infty(R^d)$ for every $s\in[0, \infty)$. We will show the fundamental solution $P(s, t)$: $C^\infty(R^d; R^d)\to C^\infty(R^d; R^d)$ for the system of heat equations:

$$(1.2)\qquad \left(\frac{\partial}{\partial s}+A(s)\right)u = 0$$
$$u(t) = g\in C_b^\infty(R^d; R^d)$$

has a C^∞-density function if

(1.3) mappings $(s, x)\mapsto\partial_x^\alpha h(s, x)$, $h\in\{a_j^{im}, c_j^m, V_j^i\}$ are all bounded and con-

(1) This work was partially supported by Grand-in-Aid Science Research (No. 59740112), Ministry of Education.