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# A NOTE ON A REPRESENTATION OF A TRANSITION BY PIVOTAL MEASURE

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It is well known that a pivotal measure for a statistical structure (or experiment) plays an important role in proving the Neyman factorization theorem ([2]). In this note we give a necessary and sufficient condition for an equivalent dominating measure of a weakly dominated statistical structure to be a pivotal measure for the structure. Motivated by a transition type of equation appeared in this characterization of pivotal measure, we consider to represent some transition, which gives an equivalency of the original experiment and its sub-experiment induced by a sufficienct subfield, by pivotal measure. This gives us another characterization of pivotal measure in terms of transition.

## 1. A characterization of pivotal measure

A triplet $\mathcal{E}=(\mathcal{X}, \mathcal{A}, \mathcal{P})$ where $\mathcal{X}$ is a set, $\mathcal{A}$ a $\sigma$-field of subsets of $\mathcal{X}$ and $\mathcal{P}$ is a family of probability measures on $(\mathcal{X}, \mathcal{A})$ is refered to as a statistical structure or an experiment synonymously. An experiment $\mathcal{E}=(\mathcal{X}, \mathcal{A}, \mathcal{P})$ is called a majorized experiment ([2] and [7]) if there exists a measure $m$ on $(\mathcal{X}, \mathcal{A})$ such that each $P$ in $\mathcal{P}$ has density w.r.t. $m$. Such a measure $m$ will be refered to as a dominating measure for $\mathcal{E}$. If a majorized experiment has a dominating measure which is localizable it is called weakly dominated. For definition of a localizable measure we refer to [2]. A special dominating measure $n$ for $\mathcal{E}$ is called a pivotal measure for $\mathcal{E}$ ([7]) if the following conditions are satisfied:

(a) $\mathcal{P}$ is equivalent to $n$, denoted by $\mathcal{P}\sim n$, namely $n(A)=0$ is equivalent to $P(A)=0$ for all $P\in\mathcal{P}$,

(b) a sub-$\sigma$-field (subfield for short) $\mathcal{B}$ of $\mathcal{A}$ is pairwise sufficient and contains supports ([2]) if and only if each $P$ in $\mathcal{P}$ has a $\mathcal{B}$-measurable version of the density w.r.t. $n$.

Here let us note that if $\mathcal{E}$ is weakly dominated and $n$ is an equivalent dominating measure for $\mathcal{E}$, $n$ is pivotal for $\mathcal{E}$ if and only if for each sufficient subfield $\mathcal{B}$ for $\mathcal{E}$ each $P$ in $\mathcal{P}$ has a $\mathcal{B}$-measurable version of the density w.r.t. $n$ (Theorem 2 in [2]). And also we note that any equivalent dominating measure $n$ has the finite subset property, that is, any $A\in\mathcal{A}$ such that $n(A)>0$ includes $B$ satisfying