Kobayashi, T.
Osaka J. Math.
**22** (1985), 289–297

# HOMEOMORPHISMS OF 3-MANIFOLDS AND TOPOLOGICAL ENTROPY

Dedicated to Professor Itiro Tamura on his 60th birthday

TSUYOSHI KOBAYASHI

(Received June 21, 1984)

## 1. Introduicton

The *topological entropy* $h(f)$ of a self-map $f$ of a metric space is a measure of its dynamical complexity (for the definition of topological entropy see section 2 below). In [$T_1$] Thurston has shown that any homeomorphism $f$ of a compact hyperbolic surface is isotopic to $\varphi$ which is either periodic, pseudo-Anosov, or reducible (see also [F-L-P], [H-T], [M]). We call $\varphi$ *Thurston's cannonical form* of $f$. In section 2 we show that $h(\varphi)\leq h(f)$ i.e. $\varphi$ attains the minimal entropy in its isotopy class. Hence from the dynamical viewpoint Thurston's cannonical form plays an important role ([H], [K], [Smi]).

In this paper, we find a similar cannonical form of a homeomorphism of a class of geometric 3-manifolds (for the definition and fundamental properties of geometric 3-manifolds see [Sc]). We note that every self-homeomorphism of a hyperbolic 3-amnifold is homotopic to a periodic one ([Mo]). In the following, we consider homeomorphisms of an $H^2\times R$, $\widetilde{SL_2(R)}$, or Nil 3-manifold $M$.

Then our main result is:

**Theorem 2.** *Let $f$ be a homeomorphism of an $H^2\times R$, $\widetilde{SL_2(R)}$, or Nil 3-manifold $M$. Then $f$ is homotopic to $\varphi$ such that either*:

(i) *$\varphi$ is of type periodic,*

(ii) *$\varphi$ is of type pseudo-Anosov, or*

(iii) *there is a system $\mathcal{T}$ of tori in $M$ such that $\varphi$ is reducible by $\mathcal{T}$. There is a $\varphi$-invariant regular neighborhood $\eta(\mathcal{T})$ of $\mathcal{T}$ such that each $\varphi$-component of $M-Int\ \eta(\mathcal{T})$ satisfies* (i) *or* (ii). *Each component $\eta(T_j)$ of $\eta(\mathcal{T})$ is mapped to itself by some positive iterate $\varphi^{m_j}$ of $\varphi$ and $\varphi^{m_j}|_{\eta(T_j)}$ is a twist homeomorphism.*

For the definitions of the terms which appear in Theorem 2, see section 4 below. We note that if $M$ is sufficiently large, then $\varphi$ is isotopic to $f$ ([Wa]).

In section 5 we show that the above $\varphi$ attains the minimal entropy in its homotopy class, and $h(\varphi)$ is positive if and only if $\varphi$ contains a component of type pseudo-Anosov.