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ON COMPLETE KÄHLER MANIFOLDS WITH FAST CURVATURE DECAY

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0. Introduction

We call (M, o) a Riemannian manifold with a pole iff M is a Riemannian manifold and $\exp_o: T_o M \to M$ is a global diffeomorphism. We define the radial curvatures at $x \in M$ as the sectional curvatures of all the 2-dimensional planes in $T_x M$ which are tangent to the unique geodesic joining the pole o to x, and write r(x) for the distance function from o. Suppose now our (M, o) satisfies the following conditions:

(0.1) There exist C^{∞} functions k, K: $[0, \infty) \rightarrow [0, \infty)$ such that

1.
$$-K(r(x)) \leq \text{all the radial curvatures at } x \leq k(r(x))$$
,

2.
$$\int_0^\infty sK(s)ds < \infty$$

3.
$$\int_{a}^{\infty} sk(s) ds \leq 1$$

In this paper, we shall prove the following theorem.

Main Theorem. Let (M, o) be an n-dimensional complete Kähler manifold with a pole o satisfying condition (0.1) $(n \ge 2)$. Moreover assume that there exists a C^{∞} function $H: [0, \infty] \rightarrow [0, \infty)$ such that

$$(0.2) \qquad \int_0^\infty s H(s) ds < \infty ,$$

and

$$(0.3) \qquad -H(r(x)) \leq tha \ Ricci \ curvature \ at \ x \leq H(r(x)) \ .$$

Then there exists a positive constant γ_0 depending only on K(s) such that if

$$\int_0^\infty sk(s)ds < \gamma_0$$
,

M is biholomophic to C^{n} .

It was conjectured by Greene and Wu that if M is an n-dimensional com-