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RIEMANNIAN SUBMERSIONS OF SPHERES WITH TOTALLY GEODESIC FIBRES

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Introduction

Let M and B be C^∞ Riemannian manifolds. By a *Riemannian submersion* we mean a C^∞ mapping $\pi: M\to B$ such that π is of maximal rank and π_* preserves the lengths of horizontal vectors, i.e., vectors orthogonal to the fibre $\pi^{-1}(x)$ for $x\in B$.

In his paper [7] Richard H. Escobales, JR dealt with the problem of classifying (upto equivalence) the Riemannian submersions of standard spheres $S^n(1)$ of unit radius in the Euclidean space $\boldsymbol{R}^{n+1}$ onto various Riemannian manifolds B under the assumption that the fibres are *connected and totally geodesic.* He proved (Theorem 3.5 [7]) that as a fibre bundle, the Riemannian submersion $\pi: S^n\to B$ is one of the following types:

a) $S^1 \dashrightarrow S^{2n+1} \downarrow \boldsymbol{C}P^n \quad n\geq 2$

b) $S^3 \dashrightarrow S^{4n+3} \downarrow \boldsymbol{H}P^n$ for $n\geq 2$

c) $S^1 \dashrightarrow S^3 \downarrow S^2(\frac{1}{2})$

d) $S^3 \dashrightarrow S^7 \downarrow S^4(\frac{1}{2})$

e) $S^7 \dashrightarrow S^{15} \downarrow S^8(\frac{1}{2})$

Further in cases (a) and (b), B is isometric to complex and quaternionic projective space respectively of sectional curvature K^* with $1\leq K^*\leq 4$. In cases (c), (d), (e), B is isometric to a sphere of curvature 4 as indicated in the diagram.

He also proved uniqueness in the cases (a), (b), and (c) but left the cases (d)