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PARALLEL SUBMANIFOLDS IN A QUATERNION PROJECTIVE SPACE

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1. Introduction

We call submanifolds with parallel second fundamental form parallel submanifolds. It is well-known that parallel submanifolds in Riemannian locally symmetric spaces are locally symmetric. It is an interesting problem to classify the parallel submanifolds in a specific Riemannian symmetric space. In fact, these submanifolds have been classified by several authors when the ambient spaces are real space forms and complex space forms (for reference see remark after Theorem 3.10). In this paper we shall classify parallel submanifolds in a quaternion projective space and its non-compact dual.

A quaternion projective space and its non-compact dual are quaternionic Kaehler manifolds. After recalling these notions, we define in §2 three kinds of immersions of a Riemannian manifold into a quaternionic Kaehler manifold, namely, totally real, totally complex, and invariant immersions, and then study fundamental properties of these immersions. In §3, we shall show that a parallel but not totally geodesic submanifold in a quaternion projective space or its non-compact dual is one of the following (Theorem 3.10):

(R–R) totally real submanifold which is contained in a totally real totally geodesic submanifold,

(R–C) totally real submanifold which is contained in a totally complex totally geodesic submanifold,

(C–C) totally complex submanifold which is contained in a totally complex totally geodesic submanifold,

(C–H) totally complex submanifold which is contained in an invariant totally geodesic submanifold whose dimension is twice the dimension of the submanifold.

It is known that a totally real totally geodesic submanifold and a totally complex totally geodesic submanifold of a quaternion projective space or its non-compact dual are a real space form and a complex space form respectively. Therefore

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