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INVOLUTIONS ON TORUS BUNDLES OVER S^1

Dedicated to the memory of Professor Takehiko Miyata

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Introduction

Involutions on torus bundles have been studied by several authors [10, 11, 13, 15, 17, 20, 22, 25]. In particular, involutions on $S^1 \times S^1 \times S^1$ and orientation-reversing involutions on orientable torus bundles have been classified by Kwun-Tollefson [13] and Kim-Sanderson [10] respectively.

The purpose of this paper is to classify all involutions on torus bundles. In fact, we will give a finite procedure for finding all involutions on a torus bundle M_A form its monodromy matrix A (see Section 2). It should be noted that involutions on a given non-orientable torus bundle are not necessarily distinguished by their quotients (Example 4.7). Here the *quotient* of an involution h on a space M means the pair $(M/h, \mathrm{Fix}(h)/h)$. As a consequence of our main theorems, we obtain the following result, which sharply improves the estimates given by Kojima [11] on the number of non-equivalent symmetries on torus bundles.

Theorem. (1) *If M_A is an orientable torus bundle, then* $1 \leqq |\mathrm{Inv}(M_A)| \leqq 21$.
(2) *If M_A is a non-orientable torus bundle with $tr(A) \neq 0$, then* $1 \leqq |\mathrm{Inv}(M_A)| \leqq 7$.

Here $\mathrm{Inv}(M_A)$ denotes the set of all equivalence classes of involutions on M_A, and $|S|$ denotes the cardinality of S. The following examples show that the above estimates are the best possible.

EXAMPLE. (1) If $A=\begin{bmatrix} 40 & 9 \\ 31 & 7 \end{bmatrix}$, then $|\mathrm{Inv}(M_A)|=1$.

(2) If $A=\begin{bmatrix} 89 & 20 \\ 40 & 9 \end{bmatrix}$, then $|\mathrm{Inv}(M_A)|=21$.

(3) If $A=\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, then $|\mathrm{Inv}(M_A)|=1$.

(4) If $A=\begin{bmatrix} 21 & 4 \\ 16 & 3 \end{bmatrix}$, then $|\mathrm{Inv}(M_A)|=7$.

As an application, a simple sufficient condition for a torus bundle to have

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