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ON THE CENTRALIZER OF THE LAPLACIAN OF $P_n(C)$ AND THE SPECTRUM OF COMPLEX GRASSMANN MANIFOLD $G_{2,n-1}(C)$

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0. Introduction. The purpose of the present paper is two-fold. The first half of which is to determine the centralizer of the Laplacian Δ of the complex projective space $\boldsymbol{P}_n(\boldsymbol{C})$ with the Fubini-Study metric g_0 and the other is to calculate explicitly the spectrum of the Grassmann manifold $\boldsymbol{G}_{2,n-1}(\boldsymbol{C})$ with the canonically normalized invariant metric g_1, as well as to give an explicit eigenspace decomposition of the Laplacian $\Delta^\wedge$ on $C^\infty(\boldsymbol{G}_{2,n-1}(\boldsymbol{C}))$ as a complex analogue of our previous paper [5].

For this purpose we begin with some preliminaries on the algebra $\mathfrak{D}^*(\boldsymbol{P}_n(\boldsymbol{C}))$ of complex linear differential operators as well as the graded algebra $\boldsymbol{S}^*(\boldsymbol{P}_n(\boldsymbol{C}))$ (resp. bigraded algebra $\boldsymbol{S}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))$) of complex contravariant symmetric tensor fields on $\boldsymbol{P}_n(\boldsymbol{C})$.

The centralizer of Δ in $\mathfrak{D}^*(\boldsymbol{P}_n(\boldsymbol{C}))$ is determined in **2**. Theorem 2.1 asserts that it coincides with the subalgebra of $\mathfrak{D}^*(\boldsymbol{P}_n(\boldsymbol{C}))$ generated by all Killing vector fields. The Killing algebra $\boldsymbol{K}^*(\boldsymbol{P}_n(\boldsymbol{C}))$ is introduced as the graded subalgebra of $\boldsymbol{S}^*(\boldsymbol{P}_n(\boldsymbol{C}))$ generated by all Killing vector fields. We also define the Plücker algebra: $\boldsymbol{K}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))=\boldsymbol{K}^*(\boldsymbol{P}_n(\boldsymbol{C}))\cap \boldsymbol{S}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))$. In **3** the Radon-Michel transform $^\wedge$: $\boldsymbol{S}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))\to C^\infty(\boldsymbol{G}_{2,n-1}(\boldsymbol{C}))$ is introduced. It has the following remarkable properties:

(i) $^\wedge$ commutes with the Lichnerowicz operator in the sense of Theorem 3.2.

(ii) The Plücker algebra $\boldsymbol{K}^{**}(\boldsymbol{P}_n(\boldsymbol{C})$ is transformed by $^\wedge$ onto the subalgebra of $C^\infty(\boldsymbol{G}_{2,n-1}(\boldsymbol{C}))$ generated by normalized Plücker coordinates.

Theorem 2.1 enables us to obtain an eigenspace decomposition of the Lichnerowicz operator restricted to $\boldsymbol{K}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))$ (Theorem 4.1). In virtue of Theorem 3.2 the eigenspace decomposition of $\Delta^\wedge$ is obtained by transferring that of the Lichnerowicz operator in $\boldsymbol{S}^{**}(\boldsymbol{P}_n(\boldsymbol{C}))$ to $C^\infty(\boldsymbol{G}_{2,n-1}(\boldsymbol{C}))$ by means of the Radon-Michel transform (Theorem 4.2).

Finally, in the appendix we give a sequence of the differential opreators, annihilating eigenfunctions of the Laplacian Δ of $\boldsymbol{P}_n(\boldsymbol{C})$.