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ON NON-SINGULAR HYPERPLANE SECTIONS OF SOME HERMITIAN SYMMETRIC SPACES

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Let $P^k(\boldsymbol{C})$ denote a complex projective space of dimension k. The product space $P^m(\boldsymbol{C})\times P^n(\boldsymbol{C})$ has a natural imbedding in $P^{mn+m+n}(\boldsymbol{C})$, called the Segre imbedding. Let V be a non-singular hyperplane section of $P^m(\boldsymbol{C})\times P^n(\boldsymbol{C})$ in $P^{mn+m+n}(\boldsymbol{C})$. The identity connected component $\mathrm{Aut}_0(V)$ of the group of all holomorphic automorphisms of V has been determined by *J-I.* Hano [3]. For an irreducible Hermitian symmetric space M of compact type we have the canonical equivariant imbedding $j: M\to P^N(\boldsymbol{C})$. Now take a non-singular hyperplane section V of M in $P^N(\boldsymbol{C})$. In this note we shall determine the structure of the Lie algebra of $\mathrm{Aut}(V)$ fro the cases when M is a complex Grassmann manifold $G_{m,2}(\boldsymbol{C})$ of 2-planes in $\boldsymbol{C}^m$ and when M is SO(10)/U(5), by applying Hano's method. In particular, using Lichnerowicz-Matsushima's theorem, we prove the following.

1) For the case M is $G_{m,2}(\boldsymbol{C})$ ($m\geq 4$), if m is odd a non-singular hyperplane section V does not admit any Kähler metric with constant scalar curvature, and if m is even V is a kählerian C-space.

2) For the case M is SO(10)/U(5), V does not admit any Kähler metric with constant scalar curvature.

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1. Preliminaries

A simply connected compact homogeneous complex manifold is called a C-space. A C-space is said to be kählerian if it admits a Kähler metric. We recall some known facts on kählerian C-spaces and holomorphic line bundles on these complex manifolds (cf. [1], [4]).

Fact 1. *Every holomorphic line bundle on a kählerian C-space M is homogeneous. If we denote by $H^1(M, \theta^*)$ the group of all isomorphism classes of holomorphic line bundles on M and by $c_1(F)$ the Chern class of a holomorphic line bundle F, then the homomorphism $F\to c_1(F)$: $H^1(M, \theta^*)\to H^2(M, \boldsymbol{Z})$ is bijective.*

Fact 2. *Every ample holomorphic line bundle on a kählerian C-space M is*