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BLOCK INTERSECTION NUMBERS OF BLOCK DESIGNS II

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1. Introduction

Let t, v, k and λ be positive integers with $v \geq k \geq t$. A t-(v, k, λ) design is a pair consisting of a v-set Ω and a family $\boldsymbol{B}$ of k-subsets of Ω, such that each t-subset of Ω is contained in just λ elements of $\boldsymbol{B}$. Elements of Ω and $\boldsymbol{B}$ are called points and blocks, respectively. A t-$(v, k, 1)$ design is often called a Steiner system $S(t, k, v)$. A t-(v, k, λ) design is called nontrivial provided $\boldsymbol{B}$ is a proper subfamily of the family of all k-subsets of Ω, then $t<k<v$. In this paper we assume that all designs are nontrivial. For a t-(v, k, λ) design $\boldsymbol{D}$ we use $\lambda_i (0 \leq i \leq t)$ to represent the number of blocks which contain a given set of i points of $\boldsymbol{D}$. Then we have

$$\lambda_i = \frac{\binom{v-i}{t-i}}{\binom{k-i}{t-i}} \lambda = \frac{(v-i)(v-i-1)\cdots(v-t-1)}{(k-i)(k-i-1)\cdots(k-t-1)} \lambda \quad (0 \leq i \leq t).$$

A t-(v, k, λ) design $\boldsymbol{D}$ is called block-schematic if the blocks of $\boldsymbol{D}$ form an association scheme with the relations determined by size of intersection (cf. [3]). Any Steiner system $S(2, k, v)$ $(t=2)$ is block-schematic (cf. [2]). For a block B of a t-(v, k, λ) design $\boldsymbol{D}$ we use $x_i(B)$ $(0 \leq i \leq k)$ to denote the number of blocks each of which has exactly i points in common with B. If, for each i $(i=0, \cdots, k)$, $x_i(B)$ is the same for every block B, we say that $\boldsymbol{D}$ is block-regular and we write x_i instead of $x_i(B)$. Any Steiner system $S(t, k, v)$ is block-regular (cf. [6]), and any block-schematic t-(v, k, λ) design is also block-regular.

Atsumi [1] proved

Result 1. If a Steiner system $S(t, k, v)$ is block-schematic with $t \geq 3$, then $v \leq k^4 \binom{k}{\left[\frac{k}{2}\right]}$ holds.

Yoshizawa [7] extended Result 1 and prove

Result 2. (a) For each $n \geq 1$ and $\lambda \geq 1$, there exist at most finitely many block-schematic t–(v, k, λ) designs with $k-t=n$ and $t \geq 3$.