ON MAXIMAL SUBMODULES OF A FINITE DIRECT SUM OF HOLLOW MODULES III

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(Received December 6, 1983)

(Revised March 6, 1984)[1]

We have studied, in [3], a right artinian ring R satisfying Condition I (see below) as a generalization of right artinian serial rings. However, there, we have restricted ourselves to the case that $J^2=0$, where J is the Jacobson radical of R.

In this paper, instead of removing the restriction $J^2=0$, we shall add one more condition (Condition II': *every hollow module is quasi-projective* or Condition II'': *R is an algebra of finite dimension over an algebraically closed field*). We shall give a characterization of a right artinian ring satisfying Condition I and Condition II' (resp. II''), and show that such a ring is closely related to an algebra of right local type studied by H. Tachikawa [8] (see also [7]). Actually, if the assumption *"left serial"* is removed in [8], the situation is very similar to that in this paper.

Further, under Condition I, we shall consider Condition II: $|eJ/eJ^2| \leq 2$ *for each primitive idempotent e*, which is weaker than Conditions II' and II''. We shall give the structure of a ring satisfying Conditions I and II, and show that the structure gives a characterization of such a ring provided $J^3=0$.

1 Conditions and Theorems. In this paper, we shall study a right artinian ring R with identity, and every R-module is assumed to be a unitary right R-module. We denote the Jacobson radical and the socle of an R-module M by $J(M)$ and $\mathrm{Soc}(M)$, respectively. Occasionally, we write $J=J(R)$. $|M|$ means the length of a composition series of M. If eR is a right uniserial module for each primitive idempotent e, R is called a *right serial (generalized uniserial) ring*. If R is a right serial ring then the following conditions are satisfied:

Condition I: *every submodule in any finite direct sum of hollow modules is also a direct sum of hollow modules* [3]

and

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1) Conditions II and II-a are added in the revise.