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ON REGULAR SELF-INJECTIVE RINGS

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A ring R is called right bounded if every essential right ideal contains a nonzero two-sided ideal which is an essential ideal as a right ideal. If R is both right and left bounded, then we call R bounded. The boundedness is well studied for noetherian rings ([1]). In this paper, we study (Von Neumann) regular rings which are right bounded. This class includes all regular rings whose primitive factor rings are artinian. We know from S. Page [3] that a regular ring R is a FPF-ring if and only if R is a self-injective ring with bounded index of nilpotence. As a result, it follows that regular FPF-rings are bounded.

In Proposition 1, we shall show that right non-singular right FPF-rings are right bounded. In our main theorem (Theorem 1), we shall show that a regular right self-injective ring R is right bounded if and only if $R \cong \prod M_{n(t)}(R_t) \times \prod T_s$, where each R_t is an abelian regular self-injective ring and each T_s is a right full linear ring. In Theorem 2, as an application of Theorem 1, we shall give a necessary and sufficient condition for the maximal right quotient ring $Q(R)$ of a regular ring R to be Type I_f.

1. Preliminaries

Throughout this paper, R denotes an associative ring with identity element and we assume that all R-modules are unitary. We freely use terminologies and the results in [2].

Let R be a regular, right self-injective ring. R is called Type I if it contains an idempotent e such that eR is faithful right R-module and eRe is an abelian regular ring. R is called Type II if it contains an idempotent e such that eR is faithful right R-module and eRe is a directly finite regular ring, and R contains no nonzero idempotent f such that fRf is an abelian regular ring. R is called Type III if it contains no nonzero idempotent e such that eRe is directly finite.

Goodearl and Boyle have shown that if R is a regular, right self-injective ring, then $R=R_1\times R_2\times R_3$, where R_1 is Type I, R_2 is Type II and R_3 is Type III (Theorem 10.13 [2]). Furthermore, they have shown that if R is a regular, right