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A GENERALIZATION OF HALL QUASIFIELDS

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1. Introduction

Let $Q=Q(+, \circ)$ be a right quasifield which satisfies the following conditions:

(1.1) Q is a two dimensional left vector space over its kernel K with a basis $\{1, \lambda\}$.

(1.2) There exist two mappings r and s from $K^{\sharp}=K-\{0\}$ into K such that every element $\xi=a+b\lambda$ of Q not in K satisfies the equation $\xi^2-r(b)\xi-s(b)=0$.

(1.3) Each element of K commutes with all the elements of Q.

Several examples of such Q are known. For example, the Hall quasifields satisfy the conditions above, where r and s are constant functions and the quadratic polynomial x^2-rx-s is irreducible over K. Moreover, the quasifields which correspond to the spread sets constructed by Narayana Rao and Satyanarayana [3] also satisfy the conditions above, where $r(x)=3x^{-1}$, $s(x)=2x^{-2}$ and $K=GF(5^{2^n-1})$.

The purpose of this paper is to study the quasifields satisfying the conditions (1.1)–(1.3). In §2 we prove the following theorem which gives a condition for $Q(+, \circ)$ to be a quasifield.

Theorem 1. *Let K be a field and let r and s be mappings from $K^{\sharp}$ into K such that* (i) *$x^2-r(u)x-s(u)$ is irreducible over K for each $u\in K^{\sharp}$ and* (ii) *$v^2-r(x)v-s(x)=wx$ has a unique solution in $K^{\sharp}$ for each $v\in K$, $w\in K^{\sharp}$. Let $Q=\{x+y\lambda \mid x, y\in K\}$ be a left vector space over K. If a multiplication $\circ$ on Q is defined by*

$$(z+t\lambda)\circ(x+y\lambda) = \begin{cases} zx-ty^{-1}F(x, y)+(zy-tx+t\, r(y))\lambda & \text{if } y\neq 0\,, \\ zx+tx\lambda & \text{if } y=0\,, \end{cases}$$

where $F(x, y)=x^2-r(y)x-s(y)$, then $Q(+, \circ)$ is a quasifield which satisfies (1.1)–(1.3).

Let $K=GF(q)$ and let Φ_K be the set of the ordered pairs (r, s) such that r and s satisfy (i) and (ii) of Theorem 1. The spread set $\Sigma_{r,s}$ which corre-