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ON WEAKLY TRANSITIVE TRANSLATION PLANES

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1. Introduction

Let π^{l_∞} be a translation plane of order p^r with p a prime. Let G be a subgroup of the translation complement and Δ a subset of l_∞ with $|\Delta|=p+1$. π is said to be Δ-transitive if the following conditions are satisfied (V. Jha [4]):

(i) G leaves Δ invariant and acts transitively on l_∞-Δ.

(ii) G fixes at least two points of Δ.

(iii) G has a normal Sylow p-subgroup.

On Δ-transitive planes, V. Jha has proved the following theorem.

Theorem (V. Jha [4]). *If π^{l_∞} is Δ-transitive with $|\Delta|=p+1$, then π has order p^2 and $\Delta=\pi_0\cap l_\infty$ where π_0 is a subplane of order p.*

If $(\pi^{l_\infty}, \Delta, G)$ satisfies the conditions (i) and (ii) above, π is said to be weakly transitive.

In his paper [4], V. Jha has conjectured that weakly transitive planes are the Hall planes of order p^2, the Lorimer-Rahilly plane of order 16 and the Johnson-Walker plane of order 16.

In this paper we prove the following theorems on weakly transitive planes.

Theorem 1. *Let π^{l_∞} be a translation plane of order p^r with p a prime and Δ a subset of l_∞ with $|\Delta|=p+1$. If a subgroup G of the translation complement of π leaves Δ invariant and acts transitively on l_∞–Δ, then one of the following holds.*

(i) *$O_p(G)$ is semiregular on Δ–$\{A\}$ for some point $A\in\Delta$.*

(ii) *π has order p^2.*

(iii) *π has order p^3 and G is transitive on Δ.*

The Lorimer-Rahilly plane of order 16 and the Johnson-Walker plane of order 16 are examples of the case (i). The Hall planes of order p^2 and the plane of order 25 constructed by M.L. Narayana Rao and K. Satyanarayana in [6] are examples of the case (ii). The desarguesian plane of order 27 is an example of the case (iii).

As an immediate corollary we have the following.