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ON QUASIFIELDS

Dedicated to Professor Kentaro Murata on his 60th birthday

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1. Introduction

A finite translation plane Π is represented in a vector space $V(2n, q)$ of dimension $2n$ over a finite field $GF(q)$, and determined by a spread $\pi=\{V(0), V(\infty)\} \cup \{V(\sigma) \mid \sigma \in \Sigma\}$ of $V(2n, q)$, where Σ is a subset of the general linear transformation group $GL(V(n, q))$. Furthermore Π is coordinatized by a quasifield of order q^n.

In this paper we take a $GF(q)$-vector space in $V(2n, q^n)$ and a subset Σ^* of $GL(n, q^n)$, and construct a quasifield. This quasifield consists of all elements of $GF(q^n)$, and has two binary operations such that the addition is the usual field addition but the multiplication is defined by the elements of Σ^*.

2. Preliminaries

Let q be a prime power. For $x \in GF(q^n)$ put $x=x^{(0)}$, $\bar{x}=x^{(1)}=x^q$ and $x^{(i)}=x^{q^i}$, $i=2, 3, \cdots, n-1$. Then the mapping $x \to x^{(i)}$ is the automorphism of $GF(q^n)$ fixing the subfield $GF(q)$ elementwise.

For a matrix $\alpha=(a_{ij}) \in GL(n, q^n)$ put $\bar{\alpha}=(\overline{a_{ij}})$. Let

$$\omega=\begin{pmatrix} 0 & \cdots & \cdots & 0 & 1 \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}$$

be an $n \times n$ permutation matrix. Set $\mathfrak{A}=\{\alpha \in GL(n, q^n) \mid \bar{\alpha}=\alpha\omega\}$.

Lemma 2.1. *$\mathfrak{A}=GL(n, q)\alpha_0$ for any $\alpha_0 \in \mathfrak{A}$. Furthermore let α be an $n \times n$ matrix over $GF(q^n)$. Then $\alpha \in \mathfrak{A}$ if and only if*

$$\alpha=\begin{pmatrix} a_0 & a_0^{(1)} & \cdots & a_0^{(n-1)} \\ a_1 & a_1^{(1)} & \cdots & a_1^{(n-1)} \\ \vdots & \vdots & & \vdots \\ a_{n-1} & a_{n-1}^{(1)} & \cdots & a_{n-1}^{(n-1)} \end{pmatrix}$$